Stocks and Flows in International Monetary Analysis

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I. *Introduction*

The model of international capital movements as a stock-adjustment phenomenon has become common in the international economics literature since the mid-1960s. This view was implicit in the early work of Mundell (1963), and has since become explicit in the work of Khouri and Porter (1972), Girton and Henderson (1973, 1974), and Herring and Marston (1973), to cite only a few examples. Early econometric work using the stock-adjustment view took income and interest rates as given and determined capital movements endogenously. Examples of this are Branson (1970), and the studies cited there. More recent studies, both theoretical and empirical, have included interest rates as endogenously determined, with the "real" sector of the economy and monetary policy variables taken as exogenous. Examples of this are the two-author papers cited just above. In this paper I build the stock-adjustment view into a simple theoretical model with the "real" sector endogenous, studying flow and stock equilibria in an ISLM model with foreign assets.

At any point in time, the allocation of a given wealth portfolio across foreign and domestic assets is a stock equilibrium problem. If a variable exogenous to this problem changes, it creates a stock adjustment as the portfolio is rebalanced. In addition, the change may alter the rate of accumulation of foreign assets, which is the allocation of saving to these assets. Thus a change in an exogenous variable will have a "stock-shift"

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effect as existing portfolios are rebalanced, and a "continuing-flow" effect, as the flow of saving into foreign assets is altered. Most of the existing literature has studied the stock-shift effect; there has been little analysis of the continuing-flow effect, presumably because the flow of saving depends importantly on the real sector. The signs of the two effects are not necessarily the same, as this paper shows. For example, an increase in "the" foreign interest rate (or world rate in a small-country model) will cause a stock-shift capital outflow as domestic portfolio balancers buy foreign securities. But the subsequent rise in domestic interest rates may reduce income and saving sufficiently that the continuing rate of accumulation of foreign assets is reduced. This is the type of paradoxical result that appears when the "real" sector is endogenized.

In section II below I describe a three-asset "textbook" ISLM model with trade and a fixed exchange rate. I include a non-traded domestic asset, "equities," both for realism and for interest. Without any degree of freedom for domestic interest rates there is no story to tell. The instantaneous-flow equilibrium solution of the model, with all stocks held constant, is summarized in Figure 1 and Table 4 of section III, which shows the effects of changes in exogenous variables on instantaneous-equilibrium values of the domestic interest rate r_k, income y, and holdings of foreign securities S. These are stock-shift effects.

Continuing-flow effects on capital movements, and implications for the balance of payments in instantaneous equilibria are discussed in section IV. Table 6 in section IV summarizes stock-shift and continuing-flow effects of monetary and fiscal policy. In section V the movement of the instantaneous-flow equilibrium to a full stock equilibrium is discussed. This section is an extension of the results in Blinder-Solow (1974) and Branson (1974b) to an economy with trade and capital movements. Finally, in section VI extension to a flexible-rate system is discussed.

The principal results of the analysis are the following.

1. With a non-traded asset in the system, monetary policy in an open economy has an effect on the rate of return on domestic assets in the short run, and can be used to stimulate or depress output.

2. An increase of a given amount ΔH in the monetary base by the Central bank in an open economy leads to a smaller increase in holdings of foreign securities in the short run: $\Delta S < \Delta H$. Thus not all of the monetary expansion flows out through the balance of payments.

3. With the real sector endogenous *and* a negative transactions demand for foreign securities, the *sign* of the stock-shift effect of monetary policy on holdings of foreign securities is in principle not clear, but the sign of the continuing-flow effect is clear.

4. An expansionary fiscal policy action will yield a stock-shift capital inflow (sales of S), but the sign of the continuing-flow effect is unclear.

The model studied here has a long-run stationary equilibrium and a constant (exogenously fixed at 1.0) price level with real output y free to adjust along an infinitely elastic supply curve. This corresponds to

Friedman's (1970) characterization of "the" Keynesian model.[1] I would rather describe it as an analysis of equilibrium on the demand side of the economy with a given price level. A further step in the analysis would be to introduce a supply side and price flexibility, and to study a full equilibrium model. It would be trivial to assume a positively sloped supply schedule $P = P(y)$, $P' > 0$ and go through the results obtained here adding price terms and then substituting them out using $dP = P'dy$. On the other hand, introducing a complex price expectations mechanism would greatly complicate an already difficult problem.[2]

II. *A Three-Asset Model with Trade and Capital Movements*

The model studied in this paper begins with a "textbook" ISLM specification with asset equilibrium (LM) equations that extend the analysis of Metzler (1951) and Tobin (1969) to include a foreign asset, and a commodity-flow (IS) equation that includes foreign trade. There are three assets in the model: money M, capital K, and foreign securities S.[3] Since I want to discuss open-market operations in this framework, I follow Metzler in assuming that there exists a stock of equities that are the ownership rights to the capital stock K, and that the ownership of this capital stock is distributed between the private sector and the government. The government owns K^g, leaving $K - K^g$ owned by the private sector. Then open-market operations are a swap between privately owned assets $K - K^g$ and money M, with an expansionary open-market operation being an increase in M and an increase in K^g, and with privately owned equities decreasing by the change in K^g.

Asset Equilibrium — the LM Sector

In the model, domestic asset-holders have demand functions for three imperfectly substitutable assets: K (equities), S (foreign securities) and M (money). Each of these symbols represents the nominal (= real) value of the stock of these assets in private portfolios.[4] The sum of these asset values is private-sector wealth, W. The demand for each asset is a function of the rates of return r_k and r_s, and income y. The latter is included

[1]See Dornbush (1973) for an analysis of a full-employment model that is roughly analogous to the one discussed here.

[2]It is interesting, though, to put a common trend given by the growth of the labor force plus productivity into the system and to reinterpret the results in terms of deviations from that trend. This is done in Branson (1972, part VI) using the wage-price inflation model sketched by Warren Smith (1970) and analyzed by Robert J. Gordon (1969, 1971).

[3]A list of symbols and their definitions is given in Table 1.

[4]I am ignoring capital gains, whose inclusion would complicate the analysis without adding anything to the results. For a similar model with capital gains, see Branson (1972, section III).

to represent the transactions demand for money, which I assume has a fixed zero rate of interest. The demand functions can be written as:

$$\frac{K - K^g}{W} = F^k (r_k, r_s, y) ; \tag{1}$$

$$\frac{S}{W} = F^s (r_k, r_s, y) ; \tag{2}$$

$$\frac{M}{W} = F^m (r_k, r_s, y) . \tag{3}$$

Table 1

Symbols and Definitions

Assets

- K — real capital stock measured at production cost.
- K^g — capital stock owned by the government.
- M — stock of outside money in constant dollars.
- S — stock of domestically held foreign securities.
- W — private sector wealth in constant dollars.
- r_k, r_s — financial market rates of return on equities and foreign securities.
- $q(K)$— profit rate on new investment.

Flows

- y — income and output in constant dollars.
- s — saving.
- i — real investment.
- g — government purchases.
- x — exports.
- m — imports.
- ϕ — tax function.
- e — exchange rate in units of domestic currency per unit of foreign exchange.

Operators

$\dot{X} \equiv dX_t/dt$; a dot over a variable denotes time rate of change.

$$f_i \equiv \frac{\delta f(X_1, \ldots, X_n)}{\delta X_i};$$

a subscript to a function denotes its partial derivative with respect to the relevant variables including functions of one variable.

All stock and flow variables have a time subscript that is omitted (assuming it is understood) in the text.

With wealth fixed, the sum $F^k + F^s + F^m = 1$. This implies that the partial derivatives of the F distribution functions with respect to each of their arguments sum to zero. Further, I assume that $f^k{}_k$ ($\equiv \delta F^k/\delta r_k$), $F^s{}_s$, and $F^m{}_y$ are positive, and all other partials are negative. Since the partials with respect to each argument must sum to zero, this means that $F^k{}_k \geqslant -F^s{}_k, -F^m{}_k$; $F^s{}_s \geqslant -F^k{}_s, F^m{}_s$; $F^m{}_y \geqslant -F^k{}_y, F^s{}_y$.

To use this model to study the questions at hand, I assume that equities K are non-traded assets, and that domestic asset-holders face an elastic supply of foreign securities S at a fixed interest rate r_s. Domestic asset-holders must hold the given stock of K but are free to trade between M and S. Any purchases of S implicitly reduce domestic reserves by that amount. These assumptions yield the two-part wealth, or balance sheet, constraint:

$$W = K - K^g + H \tag{4}$$

where

$$H = M + S. \tag{5}$$

One can imagine domestic asset-holders having an initial endowment of K and H, and then trading some of H for S to equilibrium.

Substitution of (2) and (3) into (5) yields

$$H = W \cdot F^s (r_k, r_s, y) + W \cdot F^m (r_k, r_s, y) . \tag{6}$$

The two portfolio balance equations (1) and (6) are constrained by (4), so they contain only one independent equation determining r_k, given K, K^g, H, r_s, and y. The discussion below focuses on (1).[5]

With the foreign securities rate fixed in the world market (the small country assumption), the financial sector is recursive. Equation (6) or (1)

[5]Here Walras' Law is being applied separately across the stock-equilibrium equations. This is appropriate in a continuous-time model in which reallocations of existing stocks can take place instantaneously, but accumulation (saving) takes time. On the separate application of Walras' Law to the stock and flow equations, see May (1970).

can be solved for r_k, and then equation (2) can be solved for S, with no feedback onto r_k.

To study the effects of changes in K, K^g H, r_s and y on r_k in the financial sector, totally differentiate (1) to obtain the LM curve:

$$\text{LM: } WF^k{}_k\, dr_k = (1 - F^k)(dK - dK^g) - F^k dH - WF^k{}_s\, dr_s - WF^k{}_y\, dy. \tag{1'}$$

Equation (1') gives the results of changes in each of the right-hand variables holding the others constant, on r_k. The changes in stock variables K, K^g and H are "helicopter" changes, including wealth effects. An open-market operation, holding wealth constant, is represented by $dK^g = dh$.

These "helicopter" changes in wealth must be interpreted carefully in the analysis that follows. They are not events that could occur instantaneously in any actual economy that includes the usual accounting identities. For example, K cannot increase except by an integral of saving and investment, and H here cannot increase except through an integral of government deficit. The "helicopter" changes are thought experiments in which we compare, strictly, speaking, two economies that are exactly alike in structure and in the value of all but one exogenous variable, at one point in time, to see the effect of a difference in the one exogenous variable. Since the differentiated system formed below in equation (8) is linear, the helicopter changes can be viewed as separating the effects of two sides of an asset exchange, as in the analysis of open-market operations. In addition, the effects of the helicopter changes in Table 4 below are an important part of the analysis of the stock equilibrium system discussed later in Section V.

The coefficient of dr_k is positive, as is the coefficient of dy, giving the positive slope of the LM curve in Figure 1. The coefficients of dK and dr_s are positive, and that of dH is negative. These give the direction of the effect of a change in each variable, shifting the LM curve in Figure 1 (+ is up).

The net effects of changes in K, K^g, H, r_s and y on holdings of foreign securities S can be obtained by totally differentiating (2) and substituting (1') for dr_k:

$$dS = \left[F^s + (1 - F^k)\frac{F^s{}_k}{F^k{}_k}\right](dK - dK^g) + \left[F^s - F^k\frac{F^s{}_k}{F^k{}_k}\right] dH + W\left[F^s{}_s - \frac{F^k{}_s F^s{}_k}{F^k{}_k}\right] dr_s + W\left[F^s{}_y - F^k{}_y\frac{F^s{}_k}{F^k{}_k}\right] dy. \tag{2'}$$

The coefficient of K depends on substitutability among K, S and M in portfolios. The condition is

$$\frac{dS}{dK} \gtreqless 0 \text{ as } \frac{S}{S+M} \gtreqless -\frac{F^s{}_k}{F^k{}_k}$$

The coefficient of dr_s can be rewritten as $(F^s{}_s F^k{}_k - F^s{}_k F^k{}_s)/F^k{}_k$. This is positive because $F^s{}_s > -F^k{}_s$ and $F^k{}_k > -F^s{}_k$. The coefficient of dy is negative; to increase transaction balances, foreign securities are sold.

The coefficient of dH in equation (2') is between zero and unity. This coefficient can be rewritten as

$$1 > \frac{S}{W} - \frac{K}{W}\left(\frac{F^s{}_k}{F^k{}_k}\right) > 0.$$

The fraction in parenthesis is between zero and minus one, making the whole expression positive, but less than unity because $S/W + K/W = 1 - M/W \leq 1$. This is one interesting implication of this financial-sector model with non-traded asset and less-than-perfect substitutability. With y fixed, less than all of an increase in H flows into S, leaving the domestic private sector holding some increase in M as a result.

To study the effects of open-market operations, we can set $dK^g = dH$ in (1') and (2'), and $dK = dr_s = dy = 0$. The result for r_k is

$$\left.\frac{dr_k}{dH}\right|_{dK^g = dH} = -\frac{1}{WF^k{}_k} < 0.$$

For S we have

$$\left.\frac{dS}{dH}\right|_{dK^g = dH} = -\frac{F^s{}_k}{F^k{}_k} < 1.$$

The effect on S of an open-market increase in H is less than unity but may be greater or less than the earlier effect of a "helicopter" increase of the same amount.

The effects of changes in the variables exogenous to the financial sector on r_k and S from equations (1') and (2') are summarized in Table 2. Note that there the S effects are *shifts* in a given stock of wealth as a response to changes in exogenous variables — one-shot capital movements.

Table 2

Effects on r_k and S in the LM Sector

Endogenous Variable	Exogenous Variables					
	y	K	K^g	H	r_s	dK^g=dH
r_k	+	+	—	—	+	—
S	—	?	?	1>>0	+	1>>0

Commodity-flow Equilibrium: The IS Sector

The LM sector yields equation (1) in the variables r_k and y, for given values of K, K^g, H, and r_s. The other equation jointly to determine r_k, y is the familiar commodity-market flow equilibrium condition:

$$s(y - \phi(y), W) + \phi(y) + m(y, e) = i(r_k, K) + g + x(e) \qquad (7)$$

A few comments on the functions in (7) may be useful. The saving function makes saving dependent on disposable income and wealth with $s_y > 0$ and $s_w < 0$. I assume that the government returns its dividend earnings to the public as a lump-sum taxable transfer, so that the tax base is national income. The investment function represents producers' decisions to buy additional capital goods depending on the relation between the profit rate on new investment q(K) and the rate of return r_k that they are required to pay savers to lodge equity issues in their portfolios as in equation (1). Thus the investment function is given by

$$i = i(q(K) - r_k); \quad i(0) = 0; \quad i' > 0.$$

Here the profit rate on investment q is a decreasing function of K so that

$$\frac{\delta i}{\delta K} = i'q' < 0, \text{ and } \frac{\delta i}{\delta r_k} = -i' < 0.$$

Thus the investment function can be rewritten in the form.

$$i = i(r_k, K); \quad i_{rk} = i' < 0$$
$$i_k = i'q' < 0.$$

Exports depend on foreign demand and the exchange rate e, which is given in home currency per unit of foreign exchange so that $x_e > 0$. Imports depend on income and the exchange rate; $m_y > 0$ and $m_e < 0$.[6]

We can think of the IS equation as determining y, given values for the other variables. Substitution of the wealth definition from (4) into the saving function, and total differentiation of (7) yields the IS equation.

$$\text{IS: } dy\,[s_y(1-\phi_y) + \phi_y + m_y]dy = i_r dr_k - s_w dH$$
$$+ (i_k - s_w)dK + s_w dK^g + dg + (x_e - m_e)de.$$

The coefficient of dy is positive, and that of dr_k is negative, giving the slope of the IS curve of Figure 1. The signs of the other coefficients are

[6] The sign of m_e implicitly assumes that the price-elasticity of import demand is greater than unity in absolute value. If this is the case, an increase in the exchange rate will result in a drop in home-currency expenditure on imports, with the drop in import quantity dominating the terms-of-trade effect.

summarized in Table 3, giving the direction of shift of the IS curve (+ is out) when one of the variables changes. The only uncertainty is the effect of an increase in the stock of capital on equilibrium y, given r_k. An increase in K tends to reduce investment directly by reducing the marginal productivity of capital relative to r_k and it also tends to reduce saving due to the increase in private-sector wealth; the balance is unclear. An increase in K^g holding K constant, reducing privately owned equities, has only the negative wealth effect, stimulating saving.

Up to now, I have not discussed the balance of payments or the exchange rate regime in any detail. This discussion is postponed to section IV, after the internal short-run equilibrium is discussed in section III. For the moment, I assume a fixed exchange rate, with sufficient reserves to finance deficits. The alternative of flexible rates will be discussed in section VI.

Table 3

Effects on y in IS Sector

Endogenous Variable	Exogenous Variables						
	r_k	K	K^g	H	g	e	dK^g=dH
y	—	?	—	+	+	+	0

III. *Instantaneous Flow Equilibrium with a Fixed Exchange Rate*

The IS equation and the LM equation determine equilibrium values for r_k and y given H, K, K^g, r_s, g, and e, as shown in Figure 1. The r_k, y combination, in turn, determines the stock of domestically held foreign securities through (2). Following Blinder-Solow (1974), we can interpret y(t), r_k(t) as an *instantaneous moving equilibrium.* The equations (1) and (7) adjust r_k and y to values where (a) savers are willing to hold existing stocks of assets in their portfolios, and (b) desired saving plus imports plus tax revenue equals desired investment plus exports plus exogenous g. This is not, of course, a long-run equilibrium position because non-zero values of s, i and $[g-\phi(y) + x(e) - m(y, e)]$ in instantaneous equilibrium continuously change wealth, the capital stock, and the composite money stock H = M + S.

To obtain the effects of changes in the stock variables, and in g and e, on instantaneous equilibrium income and interest rate the LM and IS total differentials can be combined:

$$\begin{array}{l} \\ \text{LM:} \\ \\ \text{IS:} \end{array} \overset{A}{\begin{bmatrix} -WK_y^k & -WF_k^k \\ s_y(1-\phi_y)+\phi_y+m_y & -i_r \end{bmatrix}} \begin{pmatrix} dy \\ dr_k \end{pmatrix} =$$

$$\overset{B}{\begin{bmatrix} -(1-F^k) & 1-F^k & F^k & WF_s^k & 0 & 0 \\ i_k - s_w & s_w & -s_w & 0 & 1 & x_e - m_e \end{bmatrix}} \begin{pmatrix} dK \\ dK^g \\ dH \\ dr_s \\ dg \\ de \end{pmatrix}$$

The sign patterns of the coefficient matrixes A and B are as follows:

$$\begin{bmatrix} + & - \\ a_{11} & a_{12} \\ + & + \\ a_{21} & a_{22} \end{bmatrix} \begin{pmatrix} dy \\ dr_k \end{pmatrix} = \begin{bmatrix} - & + & + & - & & \\ b_{11} & b_{12} & b_{13} & b_{14} & 0 & 0 \\ ? & - & + & & & + \\ b_{21} & b_{22} & b_{23} & 0 & 1 & b_{26} \end{bmatrix} \begin{pmatrix} dK \\ dK^g \\ dH \\ dr_s \\ dg \\ de \end{pmatrix}$$

with the a_{ij} and b_{ij} coefficients representing the entries in (8). The determinant of A is positive. Inversion of A and solution for dy, dr_k give the sign pattern of effects of changes in exogenous variable shown in Table 4. The first two rows of Table 4 gives the effects of changes in the exogenous variables on y and r_k. The third row of Table 4 gives the effects

[7]Inversion of A gives

$$\begin{pmatrix} dy \\ dr_k \end{pmatrix} = \frac{1}{|A|} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix} \begin{bmatrix} b_{ij} \end{bmatrix} \begin{bmatrix} dK \\ dK^g \\ dH \\ dr_s \\ dg \\ de \end{bmatrix}$$

for dy, dr_k.

of changes in the exogenous variables on domestic holdings of foreign securities S in the short run. Again these are "stock-shift" effects. The entries in the third row are obtained from the S total differential equation (2′), which already includes effects on S through r_k, plus the results in the y row of Table 4. As before, the system is recursive, with no feedback from S to y, r_k with H fixed. This implicitly assumes short-run sterilization of monetary flows.

Table 4

Effects of Changes in H, K, K^g, r_s, g and e on y, r_k, S in Instantaneous Equilibrium

Endogenous Variables	Exogenous Variables						
	K	K^g	H	r_s	g	e	dK^g=dH
y	?	?	+	—	+	+	+
r_k	?	—	?	+	+	+	—
S	?	?	?	+	—	—	?

Table 4 shows the effects of changes in the stock variables and in exogenously determined r_s, g and e on the short-run instantaneous equilibrium values of r_k, y, and S. The entries in the table give the signs of the results of a *comparative-static* analysis of the instantaneous flow equilibrium. This analysis changes each exogenous variable in turn and solves for the change in r_k, y and S, holding all other exogenous variables constant. This is equivalent to observing, for example, two different economies that have the same structure and history, including the same H stock, and differ only in the current value of g — the *instantaneous* rate of flow of government purchases — and determining the resulting differences in r_k and y. This is not, strictly speaking, the same as *period analysis*, in which we would ask what happens if we change g at one point in time. In that analysis we would have to change either H or K^g too, to allow for the initial impact of financing the additional government purchases.

Effects on r_k and y

The sign pattern in Table 4 follows from those in Tables 2 and 3, which give the direction of shift of the IS and LM curves of Figure 1 as the exogenous variables change. For example, an increase in H shifts LM down (— sign in Table 2) and IS out due to the wealth effect on saving

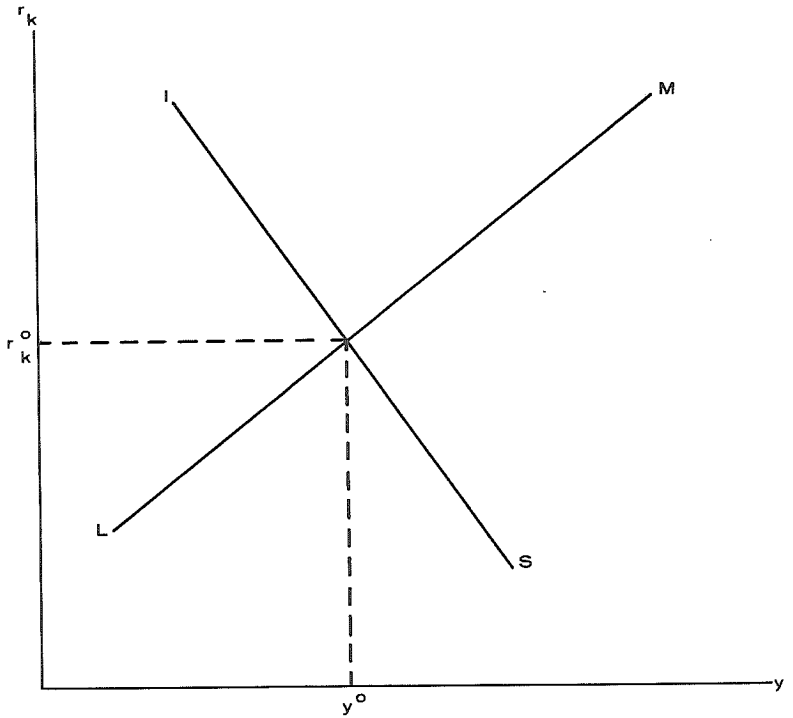

Figure 1

ISLM DETERMINATION OF INSTANTANEOUS FLOW EQUILIBRIUM r_k, y

(+ sign in Table 3). The net result is an increase in the instantaneous equilibrium value of y, but an uncertain effect on r_k. Open-market operations have a clear effect on both because they have no initial wealth effect.[8]

[8]In equation (8), setting $dK^g = dH$ and the other exogenous changes equal to zero, multiplying through B, yields

$$\begin{pmatrix} dy \\ dr_k \end{pmatrix} = \frac{1}{|A|} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix} \begin{pmatrix} dH \\ 0 \end{pmatrix}.$$

The signs of the effects of expansionary open-market operations are those of a_{22} and $-a_{21}$.

The entries for r_k and y in the K column depend on which way the IS curve is shifted by a change in K. If the saving effect were very large relative to the direct investment effect through i_k, dr_k/dK would be positive and dy/dK negative. If the direct investment effect dominated, both would be uncertain.

Stock-shift Effects on S

The S row in Table 4 gives the effects of changes in the short-run exogenous variables on the stock of foreign securities S held by the domestic private sector, given existing wealth. These differ from the S effects shown in Table 2 in one important way. Table 2 gives asset (LM) sector effects on S, holding real variables, represented by y, constant.[9] Table 4 includes the IS curve and simultaneous determination of y, allowing for additional effects on S through the transactions demand for money.

The signs of the S row in Table 4 can be obtained from equation (2') which is the S row in Table 2, combined with the y effects in Table 4. To be precise, the effect of a change in K, K^g, H, r_s, g or e on S is given by the sum of the coefficient of that variable in (2') and the inner product of the A and B matrixes in (8) that give the change in y following the change in the exogenous variable.

When any of the stock variables changes, the effect on S, *holding y constant*, is given in Table 2. But in all cases, when the IS sector is introduced so that y is allowed to vary, transactions demand effects on S (people trading S for M to increase transactions balances) appear, and sometimes work against the asset-market effects.

For an example of this complication, consider the effect of an expansionary open-market operation, $dK^g = dH$, on S in instantaneous short-run equilibrium. Earlier, we saw that holding y constant, in the asset sector

$$\left.\frac{dS}{dH}\right|_{\substack{dK^g = dH \\ dy = 0}} = -\frac{F^s{}_k}{F^k{}_k},$$

which is between zero and unity. But with y also increasing, there is an increase in the transactions demand for money, creating an offsetting pressure to sell S in order to acquire M. This is given by the product of the dy coefficient in (2') and the y effect of an open-market operation in Table 4.[10] Thus the effect on S of the open-market operation when the real sector is included is given by

[9]This is the world of Branson (1974b), Khouri-Porter (1972), Girton-Henderson (1973), and Herring-Marston (1974).

[10]From footnote 1, this effect is given by

$$\left.\frac{dy}{dH}\right|_{dK^g = dH} = a_{22} = -i_r > 0.$$

$$\left.\frac{dS}{dH}\right|_{dK^g = dH} = \overset{+}{\frac{F^s{}_k}{F^k{}_k}} + \overset{-}{\frac{W}{|A|}}\overset{+}{\left[F^s{}_y - F^k{}_y \frac{F^s{}_k}{F^k{}_k}\right]}(-i_r) .$$

The first term is the pure asset-market effect; the second is the transactions demand effect. The two work in opposite directions, and the outcome is unclear. In general, if a change in a stock variable leads to an increase in S in the asset markets alone (Table 2), but also tends to increase y once we bring in the IS sector, the net result for S is unclear. This is the source of the uncertainties about signs in the S row of Table 4.

This is another result emerging from the comparative-static analysis of the instantaneous flow equilibrium system. *If* foreign assets S have a negative income-elasticity of demand (because they are substitutes for money in transactions balances), *then* the stock-shift effect of changes in the exogenous asset variables on S becomes unclear. This ambiguity does not appear (a) if one analyzes the asset sector alone, holding y constant, or (b) if demand for S is independent of the level of income.

IV. *Capital Movements and the Balance of Payments*

In addition to the stock-shift effects on holdings of foreign assets, a change in any exogenous variable will also change the underlying *rate of accumulation* of these assets, $\dot{S}$, at any point in time. This is the amount of the current rate of saving, $s = \dot{W}$, that is going into foreign assets

The distinction between this *continuing-flow* effect and the *stock-shift* effect (see Branson, 1970) is illustrated in Figure 2. There I show the path of S through time to point t_i, when some exogenous variable is changed as in Table 4.

The *stock-shift* effect on S is given by the vertical jump from $S^0{}_i$ to $S^1{}_i$. This is the effect (assuming it is positive) in Table 4. In addition, there is an effect on the *slope* of the S path, $\dot{S} \equiv dS/dt$, at t_i. This is the difference between the dashed $\dot{S}_i$ tangents in Figure 2; it is the *continuing-flow* effect, which is the subject of this section.

The Flow of Saving into Foreign Securities

The rate of accumulation of foreign securities $\dot{S}$ is the continuing flow of saving into S. With a given S/W ratio in the asset distribution equation (2), the flow $\dot{S}$ is given by

$$S = s(y - \phi(y), W) \cdot F^s(r_k, r_s, y) ,$$

where saving $s(\cdot) \equiv \dot{W}$, the rate of growth of wealth. Thus the effect of a change in any exogenous variable on the instantaneous rate of flow $\dot{S}$ will depend on its effect on the rate of saving and on the S/W ratio, given by

$$d(S/W) = F^s{}_k\, dr_k + F^s{}_s\, dr_s.$$

These effects are summarized in Table 5.

The r_k row in Table 5 is simply brought forward from Table 2.

Table 5

Effects on the Rate of Accumulation of S in the LM Sector

Endogenous Variables	Exogenous Variables				
	K	K^g	H	r_s	$dK^g = dH$
r_k	+	—	—	+	—
S/W	—	+	+	+	+
s	—	+	—	0	0
$\dot{S}$	—	+	?	+	+

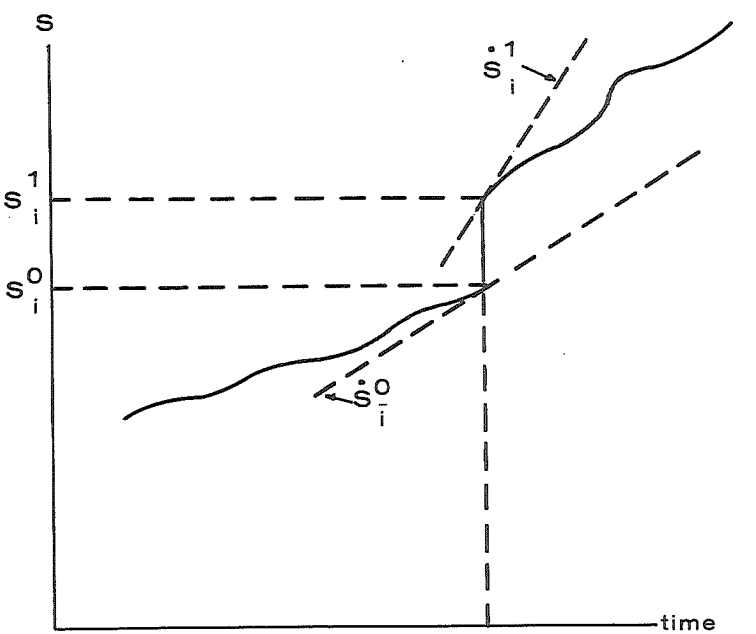

Figure 2

STOCK-SHIFT AND CONTINUING-FLOW EFFECTS ON S AND S

The S/W row comes from the expression for d(S/W) just above. The effect of a change in r_s is clear because from (2') $dr_k/dr_s < 1$, and $F^s{}_k > -F^s{}_s$. The effects on saving s are simply the wealth effects, since y is held constant. The $\dot{S}$ row just combines the two above it; the only ambiguity comes with a "helicopter" increase in H, which increases S/W but reduces the rate of saving.

Comparison of the bottom row of Table 2 which gives the stock-shift effects on S in asset markets alone, and the $\dot{S}$ continuing-flow row in Table 5, shows that the two have unambiguously the same sign in response only to changes in r_s and open-market operations. Fortunately, these are probably the most important cases. In the others — the "helicopter" changes in assets — wealth effects on s and on the denominator of S/W lead to ambiguities.

In the model specified in equations (1) — (3), adding an endogenous IS sector muddies the $\dot{S}$ results in two ways. First, changes in income affect the rate of saving. This would be the case regardless of the particular LM specification. Second, the transactions-demand substitution for money affects S/W. This is an undesirable result of the specification of (1) — (3). It is reasonable to expect an increase in y to result in a *stock-shift* sale of S to acquire M, but it is probably not reasonable to expect it to cause a decrease in the proportion of *continuing* saving going to S, as long as y stays constant at its new level. The increase in y from one level to the next should not yield a continuing accumulation of M for transactions purposes.

To avoid this implication, the asset equations could either be specified to exclude S from transactions demand altogether, eliminating the ambiguity for both S and $\dot{S}$ effects, or specified in the form

$$S/W = F^s (r_k, r_s, y/W),$$

following Tobin (1969). In a model so specified, endogeneity of y would cause $\dot{S}$ ambiguities only through the saving function.

Unfortunately, income effects on saving, when the IS sector is made endogenous, are sufficient to make the S signs of all the variables in Table 5 except open-market operations ambiguous. The first row of Table 4 gave the effects of changes in the exogenous variables on income y in instantaneous flow equilibrium. Since $\delta s/\delta y > 0$, these signs are also the income effects of changes in those variables on the rate of saving. Since the signs of K and K^g are ambiguous in Table 4, so is the direction of the effect of those variables on the rate of saving and on $\dot{S}$ in flow equilibrium. A "helicopter" increase in H increases both wealth and income, with an ambiguous effect on saving. An increase in r_s increases S/W, but by raising r_k it reduces investment, income, and the rate of saving. Thus even the sign of the effect of r_s on $\dot{S}$ is ambiguous in instantaneous flow equilibrium although the stock-shift effect $dS/dr_s > 0$.

The effect of an expansionary open-market operation on the stock of S in Table 4 was rendered ambiguous by the transactions demand substitution. If we assume that this has no effect on continuing accumulation of S, then the sign of an open-market operation on $\dot{S}$ is clearly positive; it increases the rate of saving and it increases S/W.

Two exogenous flow variables are included in Table 4 but not in Table 5: g and e. An increase in either increases both y and r_k (an outward shift in IS). Both result in a stock-shift decrease in S in Table 4. But their effects on $\dot{S}$ are ambiguous, since the rise in r_k reduces S/W, but both increase the rate of saving through y.

This is a third important point uncovered by making the stock-flow distinction in analyzing capital movements. *A fiscal-policy increase in g will result in a stock-shift inflow of capital, but its effect on the continuing rate of accumulation of foreign assets is unclear.* The unique positive effect of an expansionary fiscal policy on the capital account assumed, for example, by Fleming (1962) holds only for the stock-shift effect.

A summary of the effects of changes in monetary policy (open-market operations) and fiscal policy (changes in the rate of government purchases) on the capital account is given in Table 6. If foreign assets are substitutable for money when transactions demand changes, then the transactions effect makes the stock-shift sign of open-market operations ambiguous when the real sector (symbolized by y) is endogenous. But monetary policy has the usual effect on the *rate of accumulation* of foreign assets. On the other hand, expansionary fiscal policy yields a stock-shift capital inflow, but an ambiguous effect on the continuing rate of accumulation. Thus the use of policy instruments to influence the capital account should depend on whether one is aiming for a reserve target (stock-shifts) or a balance-of-payments targets (continuing-flow).

Table 6

Effects of Open-market Operations ($dK^g = dH$) and Fiscal Policy (dg) on S and $\dot{S}$.

Effect on	Policy Action	
	dg	$dK^g = dH$
S(stock shift)	—	?
$\dot{S}$(rate of flow)	?	+

Balance-of-Payments Effects and Reserves

With a fixed exchange rate, extension of these results to the balance of payments — the flow rate of accumulation of reserves — is easy. Expansionary monetary or fiscal policy increases the instantaneous equilibrium value of y in Table 4. This reduces the trade surplus (or increases the deficit). In Table 6, we see that the continuing-flow effect on the capital account is ambiguous in the dg case, and produces an increase in the rate of capital outflow in the monetary policy case. Thus the effect of fiscal policy on the rate of accumulation of reserves is ambiguous but the effect of monetary policy is clear. This is the familiar result from Fleming (1962) but for somewhat different reasons. It also supports the use of monetary policy for external balance defined as a zero rate of accumulation of reserves.

On the other hand, with foreign securities in the transactions demand picture, an expansionary fiscal policy gives a stock-shift increase in reserves (reduction of S), while the sign of monetary policy is ambiguous.

Finally, it is interesting to note that a shift in the monetary-fiscal policy "mix" has the same sign for both stock-shift and continuing-flow effects. Consider, for example, a combination of expansionary fiscal policy and tighter monetary policy that leaves income, and thus the trade balance, unaffected in instantaneous equilibrium. The g increase gives a stock-shift inflow, and the reduction in K^g and H reduces the continuing rate of accumulation. If the ambiguous signs in Table 6 are not attached to quantities larger than are the unambiguous signs, changes in the mix work the same way on both stocks and flows in short-run instantaneous equilibrium.

V. *Stock Adjustment and Full Stock Equilibrium*

The short-run instantaneous equilibrium system can be viewed as a mechanism that adjusts y and r_k so that *ex ante* saving and investment, and the government and trade surpluses, balance as required by the commodity-flow equilibrium condition, equation (7). This equation can be rewritten in the following form:

$$s(y - \phi(y), W) = i(r_k, K) + [g - \phi(y) + x(e) - m(y, e)] . \qquad (9)$$

The rate of change of the money stock held by the private sector, M, is the sum of the government budget deficit and the trade surplus, less purchases of foreign securities:

$$M = g - \phi(y) + x(e) - m(y, e) - S . \qquad (10)$$

On the assumption of money-finance of both the budget deficit and the balance-of-payments surplus (the Central Bank exchanges domestic H for net foreign exchange earnings at the fixed exchange rate), equation (10)

just says that M is the sum of the budget deficit and the payments surplus.[11]

Equation (9) can be rewritten as

$$s = \dot{K} + \dot{H} = \dot{K} + \dot{M} + \dot{S} . \qquad (11)$$

Saving is accumulation of new assets by the economy's consumers/portfolio balancers. Investment is the supply of new capital stock and the sum of the government and trade balances is the supply of new H, which portfolio balancers distribute between M and S.

Thus one way to view the usual commodity-flow equilibrium condition is as a mechanism that aligns savers' desired accumulation of new assets with their supply by investors, government, and the foreign sector. In the instantaneous equilibrium described above, non-zero values will be determined for these rates of change, and the resulting changes in stocks drive the instantaneous equilibrium from one point to the next. It is a moving equilibrium, hopefully moving toward a stable-stock equilibrium. Since the economy is in instantaneous equilibrium at all times, condition (11) holds at all times. Thus we can focus on the adjustment equations driving K and H, since the instantaneous-equilibrium mechanism will maintain *ex ante* saving equal to $\dot{K} + \dot{H}$. Also, if there is a stock equilibrium solution in which $\dot{K} = \dot{H} = 0$, short-run flow equilibrium will ensure that saving is also zero. The stock-adjustment equations for H and K carry the economy from a position of unbalanced portfolios to the balanced long-run growth path of the neoclassical models of Johnson (1966), Tobin (1965), and Foley-Sidrauski (1971), opened to trade and capital movements.

Table 4 above gives the signs of the partial derivatives of the *reduced-form* equations for r_k, y, and S, dependent on the exogenous values of the stock variables, r_s, g, and e. Thus r_k and y can be written as functions of these exogenous variables:

$$\begin{aligned} r_k &= h^1 (K, K^g, H; r_s, g, e) ; \\ y &= h^2 (K, K^g, H; r_s, g, e) . \end{aligned} \qquad (12)$$

These are the implicit *reduced-form* equations of the *structural model* represented by the IS and LM curves. The signs of the partial derivatives of h^1 and h^2 are the first two rows of Table 4.

With both the rate of government purchases and government holdings of equities fixed, the rate of change of H is given by the sum of the budget deficit and the trade surplus:

$$\dot{H} = P[g - \phi(y) + x(e) - m(y, e)].$$

[11]Note that here I implicitly assume no sterilization of the payments balance by open-market operations.

Since the price level is held constant throughout, it can be set at unity. With the exchange rate e fixed at its stock-equilibrium value, and substitution of the reduced-form expression for y in (12), the H stock-adjustment equation can be written as

$$\begin{aligned}\dot{H} &= g + x(e) - \phi(y) - m(y, e)\\ &= g + x(e) - \phi(h^2(K, K^g, H; r_s, g, e))\\ &\qquad - m(h^2(K, K^g, H; r_s, g, e)) .\end{aligned} \tag{13}$$

The rate of change of the capital stock (= value of equities in this model) is investment. With K^g constant, the rate of change of the capital stock held by the private sector is the same as the rate of change of the entire stock, K. The equation for K is

$$\dot{K} = i(r_k, K) = i(h^2(K, K^g, H; r_s, g, e), K), \tag{14}$$

again substituting for r_k from (12).

Given the values of the policy variables K^g and g, the fixed exchange rate e, and the foreign interest rate r_s, the two equations (13) and (14) are first-order differential equations in H and K. We can ask if this system has a stock equilibrium solution H*, K* where $\dot{H}$ and $\dot{K} = 0$, and if this solution is stable. If it is, we can study the comparative statics of the stock equilibrium, and contrast these results to the instantaneous-flow equilibrium of Table 4. These exercises are worked through in Branson (1974a), with an earlier discussion in Branson (1972). Here I will just summarize the results.

The existence of a stable long-run equilibrium in H and K is intuitively plausible. If, in an initial instantaneous equilibrium there is positive investment, this increases K and tends to raise r_k, reducing investment. If income is sufficiently low that $[g + x - \phi(y) - m(y)]$ is positive, $\dot{H}$ is positive and income rises, reducing the budget deficit and the trade surplus. Thus the system is likely to move toward the stock-equilibrium K*, H* described by setting $\dot{H}$ and $\dot{K} = 0$ in (13) and (14). In Branson (1974a), the stability by such a stock equilibrium is proved.

In stock equilibrium, from (13), $\dot{H} = 0 = g - \phi(y) + x(e) - m(y, e)$. To solve for the fixed exchange rate that is consistent with stock equilibrium, we can first set $g - \phi(y) = 0$ and solve for the y* value that balances the budget. Then with that y* value we can set $x(e) - m(y^*, e) = 0$ and solve for the e* value that balances trade. With both the budget and trade balanced in stock equilibrium, private sector portfolio-balancers allocate the fixed stock of $H(\dot{H} = 0)$ between M and S, so that in equilibrium $\dot{S} = 0$ and the balance of payments is also zero.

Since an equilibrium condition here is $g - \phi(y) = 0$, the "multiplier" between stock equilibria is just $1/\phi_y$, a result familiar from Oates (1966) and McKinnon (1969). Fiscal policy changes in g or the shape of ϕ will affect equilibrium y. Monetary policy has no effect unless it is combined with fiscal policy. Between stock equilibria, the money supply (H here) is

endogenous; the effects of open-market operations in the short run are undone by stock adjustment of the money supply. These results are by now standard in the literature on stock and flow equilibria. For more detailed analysis, see Branson (1974a).

To summarize, the stock equilibrium H*, K* is probably stable. As opposed to the usual short-run results of Table 4, between stock equilibria fiscal policy influences the level of income y and monetary policy only affects the stock of reserves. Thus in the instantaneous short-run equilibrium, open-market operations change the flow equilibrium value of the balance of payments, as well as y. When the system returns to stock equilibrium, at the original value of y, the balance of payments is again zero, but the integral of the non-zero payments balances in the interval is the effect on reserves.

VI. *Flexible Exchange Rates*

In the model described in this paper, allowing the foreign exchange market to determine the value of the exchange rate so that the balance of payments B (= $\dot{R}$, the rate of change of reserves) is zero:

$$B = \dot{R} = x(e) - m(y, e) - \dot{S} = 0 , \tag{15}$$

simply allows the exchange rate to track the instantaneous r_k, y solution to the full stock equilibrium e* value with no effect on H from external influences. To obtain a more complicated, and more realistic, result, we would have to add speculative capital movements and leads and lags on trade payments to the story. This could be done, but not in this paper.

Within the confines of the present model, capital movements (purchases or sales of foreign securities S) respond only to interest rates and transactions demands. As policy variables change, there are stock-shift and continuing-flow responses, dS and $d\dot{S}$, and as H and K move toward stock equilibrium, both responses are combined in the data being generated.[12]

In stock equilibria, a flexible rate will take on the same e* value as the fixed rate of section V. In fact, the e* from section V is the fixed-rate value that policy-makers would set by asking what value would a flexible rate take on in stock equilibrium. Thus the stock-equilibrium solutions for K*, H*, y*, e* are the same as in the fixed-rate case of section V, as shown in Branson (1974a). The additional wrinkle compared with that paper is that here private-sector portfolio-balancers can split H between M and S, where in the previous paper with no capital movements $H \equiv M$.

The difference between the fixed-rate case of sections II — V and the flexible-rate case is in adjustment out of stock equilibrium. When the economy is not in stock equilibrium, the stock of money in private sector portfolios is changing. In the fixed-rate case, the equation for $\dot{M}$ was

[12]This makes it very hard to separate stock-shift and continuing-flow responses empirically. I know of no study that has succeeded in doing so to date.

$$\dot{M} = g - \phi(y) + x(e) - m(y, e) - \dot{S},$$

with $\dot{H} = \dot{M} + \dot{S}$. When the economy is out of equilibrium in this case, both the budget deficit and the balance-of-payments surplus feed the domestic money stock.

In the flexible-rate case, e adjusts in instantaneous equilibrium to insure that

$$x(e) + m(y, e) - \dot{S} = 0.$$

In this case, when the economy is out of equilibrium, $\dot{M} = g - \phi(y)$ alone. Only the budget deficit feeds the domestic money stock. This has the implication that income will fluctuate more in the flexible-rate case then in the fixed-rate case. In the latter, a change in income affects $\dot{M}$ through both tax revenues and imports. In the flexible-rate case, income influences $\dot{M}$ only through tax revenues, so that for a given change in the money stock moving toward stock equilibrium, a bigger deviation of income from its stock equilibrium value is required.

This discussion of the flexible-rate case is necessarily sketchy. I have worked out the results fully in Branson (1974a) for a model with no foreign asset and no capital movements. But the introduction of capital movements, with the value of the exchange rate being determined by both continuing flows and discontinuous stock shifts, raises some analytical problems that are yet to be solved.

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Discussion

Giorgio Basevi

1. I divide my comments on Branson's paper into two parts. First, I shall locate what I believe are weak spots in his analysis, while keeping mine within the same overall framework and set of assumptions as Branson. Second, I shall suggest some directions toward which the model could be extended and possibly improved. I hope that on both grounds my criticism may contribute to further development and application of a line of macro-model building into which Branson's series of recent papers brilliantly fits.

2. My first comments, as just said, are kept within the same set of assumptions and analytical framework that Branson uses. Within it, the paper could be improved on four points, that I am going to present in a subjectively decreasing order of importance.

2.1. Branson's model does not incorporate into the analysis the flow of services on foreign securities (r_s S, in his notation). These should appear both in the clearing condition for the goods and services market (equation (7)), and in the equilibrium budget condition of the foreign sector (i.e., the balance of payments, equation (17)). The same omission is apparent in the equilibrium budget condition of the private sector (equation (11)). Of course, these services should be treated as export receipts when the net stock of S is positive, and as import payments when it is negative (i.e., the private sector is a net debtor vis-a-vis the foreign sector).

This assimilation to exports and imports of goods, however, is not allowed in Branson's model, where net exports (x-m) are only the function of income and the exchange rate. Adding services to the net trade account, would make it also a function of the interest rate on foreign assets (r_s) and of the other variables that determine the stock of foreign assets (r_kW). It is obvious, and well-known from the literature,[1] that this addition will introduce interesting patterns of reaction of the balance of payments to, among other things, policy-induced changes in domestic rates of

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[1]See, for example, Willett and Forte (1968).

interest, particularly with respect to the contrast between stock-shift and continuing-flow effects. It is not difficult to add this important feature to Branson's model, and I suggest that it should be done, given the emphasis he rightly puts on the distinction between the short- and long-run effectiveness of monetary and fiscal policies.

2.2 The present form of Branson's model explicitly does not take into account capital gains due to interest rate changes. This is done in another paper by Branson,[2] and it seems not to affect the conclusions significantly. There is, however, a form of capital gains (or losses) that Branson does not consider in that model, nor in this one, and that should in my opinion be included, since (contrary to the one due to interest rate changes) it introduces an additional variable into the function determining portfolio allocation. It also affects the same market clearing condition and budgetary constraints involved in the correction suggested above.

I am referring to exchange rate changes, which affect both the capital value and the flow of interest payments that originate from foreign assets, when these are denominated in foreign currency. The practical difficulty here is that S is a net credit or debit position, which results from an aggregate of assets denominated both in domestic and in (various) foreign currencies. However, in line with the small country assumption of the model, it would not be too unreal to assume that the whole of S, and its accompanying interest flows, are denominated in foreign currency. Thus, when the exchange rate (e) changes, the r_s S element to be included in equation (7), (11) and (18) should also change, and it would of course change the capital valuation of S in the portfolio allocating functions and in wealth definitions. However, this correction, which seems to me necessary for completeness, would not affect Branson's conclusions in this model, since he is not interested in the effects of devaluation, but mainly works within fixed exchange rates, or flexible rates set at the long-run level of equilibrium.

2.3. Still on capital gains, I believe that consistency should be preserved even when simplification is performed. Thus, while disregard of capital gains on S (except for those due to exchange rate changes) could be justified by assuming that S are very short-term assets, the same cannot be claimed for equities on real capital assets. The problem of market valuation of K cannot and should not be avoided, since there is no other way except through the price of an equity that the market for these assets can adjust, when, in the short run, the real stock of capital (K) and its marginal efficiency are given while r_k changes.[3]

2.4. The model is cast in terms of positive rates of saving and hence of positive $\dot{K}$, $\dot{S}$, and $\dot{M}$. Yet income (y) is supposed to stay constant through time when at the equilibrium level. It would be preferable to state clearly whether the economy is in a stationary state or on an equilibrium growth path.

[2]Branson (1972).

[3]This feature is well underlined in a recent paper by Steinherr (1973).

3. My second set of comments tries to indicate possible modifications or extensions of the model that might render it more useful for the understanding of real phenomena.

3.1. In many cases when in the model exogenous changes have ambiguous effects on endogenous variables, the culprit is the negative income elasticity of the demand for foreign assets. Branson indicates a possible alternative whereby the ratio y/W would be an argument of the allocating function. Another possibility, with perhaps more empirical content, would be to consider that while money (M) is demanded for internal transactions, foreign assets (S) are partly demanded to finance external transactions (e.g., commercial credit). This would suggest, for example, making M/W and S/W functions of the ratio between exports and income, such that

$$M/W = F^m (r_k, r_s, y, e/y)$$

$$S/W = F^s (r_k, r_s, y, e/y)$$

with $F^s_{e/y} > 0$ and $F^m_{e/y} = - F^s_{e/y}$

An alternative along the same line would be to consider that implicit in the model there are really four assets - the fourth one being foreign money (international reserves) - and to allow for a private demand for it as part of the portfolio allocation process, while these modifications might not reduce the number of ambiguous cases in the comparative statics exercises on the model, they would make more acceptable, by mitigating its effects, the idea that transactions demand for money must be satisfied by reducing holdings of foreign assets, regardless of the foreign trade flows that higher income is inducing and of the degree of openness of the economy. In fact, this approach would assume that the relative allocation of wealth between M and S (or between domestic and foreign money) is a function both of the business cycle effect on foreign trade (and its financing needs), and of the structural degree of openness of the economy.

3.2. Branson's model is the latest in the stream of contributions that are known to international-trade theorists as the "portfolio approach" to balance-of-payments theory. This approach, as Branson recalls, owes much to Tobin and has been applied to balance-of-payments analysis at the empirical no less than at the theoretical level. Branson himself has contributed much to overcoming the stock-flow controversy on capital movements with his empirical work. However, the theory of portfolio allocation promulgated by Tobin loses much of its interest if it is reduced to a world of certainty and costless adjustments. Indeed, the empirical studies that led to the "new view" of capital movements put most of their accent on two essential elements of the portfolio approach, namely the uncertainty attached to returns on various assets, and the cost of reallocating a given portfolio. This first problem is typically dealt with by making use of the mean-variance model, while the second problem is dealt with

through various schemes of partial adjustment in portfolio decisions. Obviously, introduction of these essential features of the portfolio approach would make the model cumbersome and possibly destroy the simplicity of its conclusions. I venture, however, that what would be lost in terms of definiteness would be gained in terms of empirical relevance.

3.3. Since I have taken the easy path of suggesting extensions of the model, let me conclude on a more critical tone by saying that Branson's model is disappointing for the inability, in its present form, to give answers to some of the main empirical problems currently hanging over our economies. I am referring to the problem of price inflation and the induced shifting in the allocation of wealth by portfolio holders trying to get away from money and other depreciating assets. I am also referring to the danger that a sudden crisis in the Eurocurrency market and/or the financial markets of the main industrial countries might provoke rapid deflation of some components of real wealth held by non-Government sectors, with induced effects on saving and real expenditure. Of course this would require complicating the model through the inclusion of price expectations in the wealth-allocating functions, through the enlargement of the number of assets assumed, and through the considerations already referred to under 2.2. Yet I believe that this is exactly what models like Branson's should have, since their emphasis on the importance of wealth and its allocation would be of little help if they could not be used to explain how sudden inflation and deflation of real wealth in the actual world occur and how they affect economic activity.

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