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The Uruguay Round of Trade Negotiations: Industrial and Geographic Effects in the United States

Norman S. Fieleke

No other international trade negotiations have been so comprehensive as the Uruguay Round, in which participants agreed to liberalize trade in agricultural products, to reduce tariffs on industrial products by an average of more than one-third, and to establish a World Trade Organization. This article examines the effects of the Uruguay Round agreements to liberalize trade in goods, focusing primarily on the United States.

The analysis suggests that the agreements will have only a negligible impact upon employment in nearly every U.S. manufacturing sector, in every state, and in the country as a whole. The agreed trade liberalizations (as represented by the sectoral employment changes likely to result) seem to bear little relationship to the nation's revealed comparative advantages (weighted by employment). By and large, both the United States and its trading partners apparently resisted granting sizable trade liberalizations in sectors where the other possessed a marked comparative advantage. If so, both parties will be impeded from further specializing in the sectors of their greatest comparative advantage, and world income will grow by less than if both had been more forthcoming. 3

An Assessment of the "Official" Economic Forecasts

Stephen K. McNees

While many studies have evaluated the accuracy of "official" economic forecasts, this study may be the first published analysis of the Federal Open Market Committee's "Humphrey-Hawkins" forecasts. In this article, the "official" forecasts generated by the Council of Economic Advisers (CEA), the Congressional Budget Office (CBO), and the Federal Open Market Committee (FOMC) are analyzed and compared to each other as well as to forecasts made by the private sector.

The findings for the one-year-ahead forecasts reconfirm that the CEA, CBO, and private sector forecasts are about equally accurate while more accurate than simple rules of thumb. For the multiyear real GNP forecasts, however, evidence indicates that the forecasts of prominent commercial forecasters and the CBO are more accurate than those of the CEA, owing to an optimistic bias. In addition, the midpoints of the FOMC's longest forecasts outperform one standard private sector counterpart. 13

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Stocks, Bonds, Options, Futures, and Portfolio Insurance: A Rose by Any Other Name . . . .

Peter Fortune

Trading volume and open interest in options and futures contracts on stock indices, equities, and interest rate instruments traded on world exchanges have experienced remarkable growth. However, this growth has been accompanied by controversy about the proper role of financial derivatives and the potential for abuse. Prominent attention has been given to losses by major corporations, broker-related short-term mutual funds, and municipal agencies.

The public debate about "derivatives" has promoted the impression that the heart of the problem has been a proliferation of brand new ways of making bets on future stock prices, interest rates, and exchange rates. The positive functions of derivatives as means of risk management are almost forgotten.

This article shows that exchange-traded options are really nothing new. Rather, they are repackages of the same traditional financial instruments. The article describes the practical application of the equivalence between exchange-traded options and a traditional portfolio of stocks and bonds. This is done by demonstrating the strategies of dynamic hedging and of portfolio insurance. The first uses options to hedge against stock price movements, while the second uses stocks and bonds to create "synthetic" options. 25

The Uruguay Round of Trade Negotiations: Industrial and Geographic Effects in the United States

Norman S. Fieleke

Vice President and Economist, Federal Reserve Bank of Boston. Rachel Cononi provided valuable research assistance.

History records many international trade negotiations, some trivial and some impressive, but no others so comprehensive as the Uruguay Round. As the negotiations peaked, on December 15, 1993, in Geneva, Switzerland, more countries—117 in all—reached a consensus on more issues than in any previous negotiation. Among other things, the Uruguay Round participants agreed to liberalize trade in agricultural products, to reduce tariffs on industrial products by an average of more than one-third, and to establish a World Trade Organization both to facilitate the implementation of multilateral trade agreements and to serve as a forum for future negotiations.[1]

The chief purpose of such trade agreements is to improve living standards. As trade barriers come down, countries will be stimulated to channel more of their resources into the activities they carry on relatively most efficiently in the world economy. Total exports, investment, and income will increase.

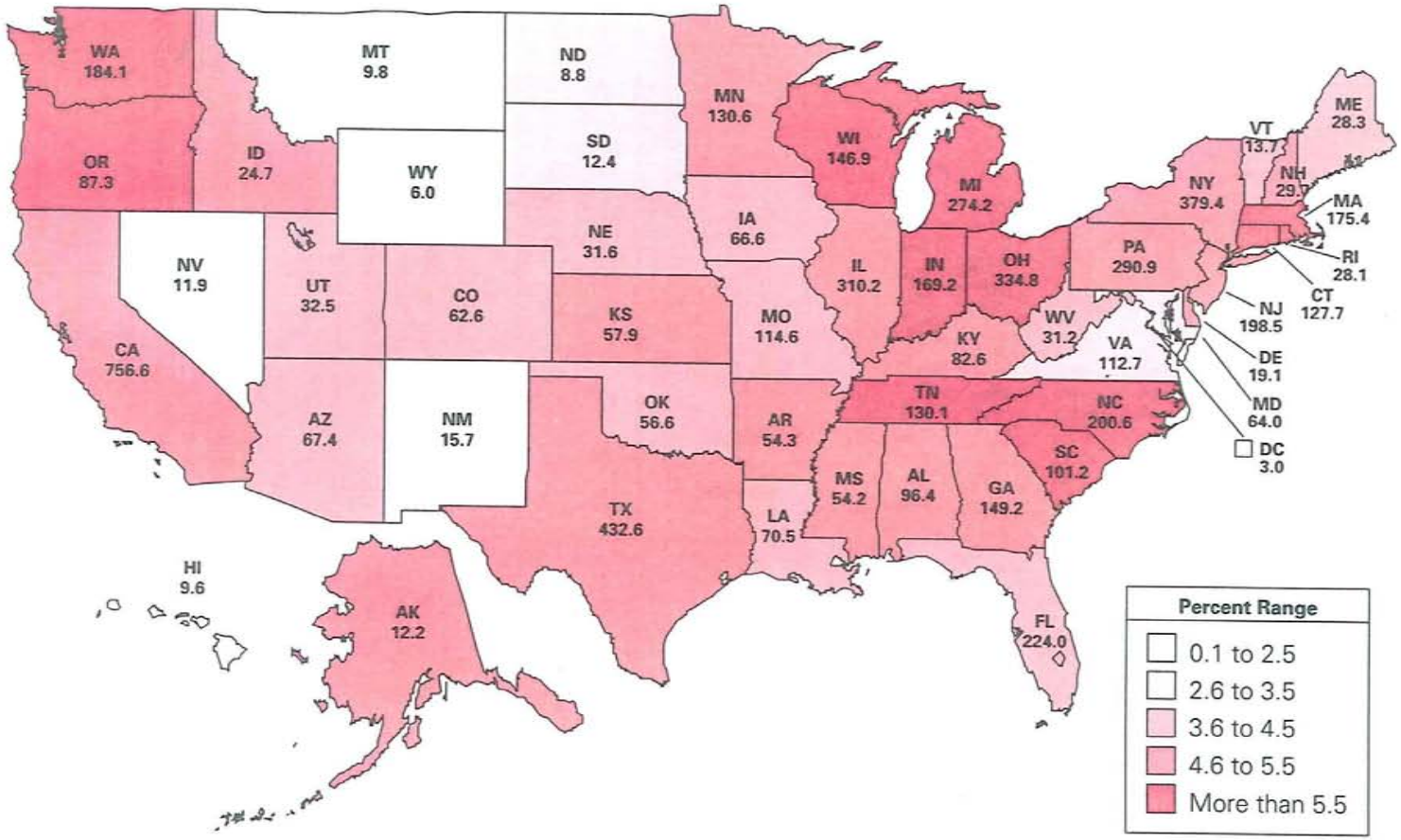
While this general outcome is widely expected from the Uruguay Round agreements, accurately quantifying their economic impact is a formidable undertaking, given their breadth and the number of countries involved. Evaluating the negotiated reductions in barriers against trade in services (such as accounting or legal services) is especially difficult. Those barriers are not readily measurable, and internationally comparable data on services imports are not available.

Fewer difficulties are encountered in appraising the agreements to liberalize trade in goods, and some fairly sophisticated estimates have been published of the effects of these agreements on world trade and income. Among the most recent and comprehensive are estimates by the Secretariat of the General Agreement on Tariffs and Trade. These estimates suggest that the agreements might raise annual world income by as much as $510 billion, measured in 1990 U.S. dollars, by the year 2005. Of this $510 billion, roughly $120 billion would accrue to the United States (GATT 1994, p. 34).

Figure 1

Employment Related to Manufactured Exports for States: 1991

(Shading indicates export-related employment as a percentage of civilian employment. Numbers are export-related employment in thousands.)



Source: U.S. Bureau of the Census, *Exports from Manufacturing Establishments: 1990 and 1991* (Washington, D.C.: 1994).

This article further examines the effects of the Uruguay Round agreements to liberalize trade in goods, focusing primarily on the United States. Following a very brief summary of the agreements, the article presents rough estimates of their impact on employment in manufacturing, both for the nation and for the individual states, and then examines how closely the estimated changes in employment correspond to the comparative advantages revealed by international trade patterns.

I. The Uruguay Round Agreements Liberalizing Trade in Goods: A Capsule Summary

Both tariff and nontariff barriers to trade are to be reduced as a result of the Uruguay Round. In general, the agreed liberalizations are to be completed by the year 2005.

For industrial products, the advanced countries committed to reduce their tariffs from an average level of 6.3 percent to 3.8 percent, and other countries also pledged noteworthy reductions. In addition, substantial decreases are to be made in nontariff barriers, which have proliferated in recent years. Prominent among these nontariff barriers are quantitative restrictions that place limits on the volume of goods—especially textiles and clothing—flowing from one country to another; these restrictions are to be relaxed considerably.

For agricultural products, the negotiators agreed, with some exceptions, to convert the substantial prevailing nontariff barriers into their tariff-equivalents, and then to lower all tariffs of advanced countries by an average of 36 percent and the tariffs of developing countries (except the least developed) by an average of 24 percent. In addition, measures were adopted to ensure that agricultural products will have access to importing countries at certain minimal levels, and significant reductions were agreed in both domestic and export subsidies.

II. Employment Effects

Many workers are engaged in the production of manufactures that the United States exports. Between 1981 and 1991 their number rose from about 4.8 million to 6.1 million, or from 4.7 percent to 5.1 percent of total civilian employment (U.S. Bureau of the Census 1983, p. 13; 1994, p. 19). As might be expected, the comparable percentage for the individual states varies widely (Figure 1).

As part of a major study of the impact of the Uruguay Round agreements, the U.S. International Trade Commission has estimated the long-term employment consequences of those agreements for the U.S. economy (U.S. ITC 1994). The estimates were presented as ranges and for sectors, or industry groups, that are defined differently from the industry groups in the Standard Industrial Classification (SIC). Our analysis converts those ranges into more specific estimates for manufacturing sectors as defined in the SIC, and also extends the estimates below the national level to the individual states.

The Uruguay Round agreements will have only a negligible impact upon employment in nearly every U.S. manufacturing sector, in every state, and in the country as a whole.

To convert the Commission's ranges into more specific estimates, we used the midpoints of those ranges. Concordances supplied by the Commission and the Census Bureau were used to allocate the sectors defined by the Commission to 2-digit SIC categories.[2] Employment changes from the Uruguay Round agreements were computed for each SIC category as the weighted (by employment) average of the percentage changes (estimated by the Commission) for the sectors allocated to the category. These percentage changes by SIC category were then assumed to apply to the individual states as well as to the nation, on the assumption that at the margin each state experienced the same intensity of foreign competition in each category.

[1] For a fuller discussion of the Uruguay Round agreements, see Norman S. Fieleke, "The Uruguay Round of Trade Negotiations: An Overview," *New England Economic Review*, May/June 1995, pp. 3–14.

[2] These allocations were inexact for some sectors which overlapped two or more SIC categories but which could not be separated into components with employment changes assignable to each category. In such cases the entire sector and its employment change were allocated to the SIC category that seemed clearly to contain the bulk of the sector's component commodities, except for miscellaneous manufacturing, for which no such judgment was feasible and which was therefore omitted from the study.

For the nation, the resulting estimated total impact on employment is remarkable, not for its enormity but for its triviality. For all manufacturing sectors combined, the Uruguay Round agreements are estimated to generate a loss of roughly 17,000 jobs, or one hundredth of 1 percent of total U.S. employment—and only over the long term. In other words, the aggregate employment impact of the agreements affecting manufacturing is likely to be approximately neutral, according to our computations based on the Commission's sector-by-sector study.[3]

Nonetheless, individual sectors could conceivably experience noteworthy impacts, with significant long-term employment gains or losses in some sectors offset by opposite changes in others. In that case, substantial frictional unemployment could occur in the short run as the labor force became redistributed. Similarly, individual states could be strongly affected.

As can be seen in Table 1, the percentage employment changes estimated for the various manufacturing sectors do indeed exceed that for the nation, but remain generally small, especially as long-term phenomena. Apparel and related products is the only sector for which a double-digit change—a loss of 10 percent—is estimated. Of the 18 sectors listed, 11 are expected to experience employment gains or losses of 0.5 percent or less. For 12 of the 18, the employment change is expected to be positive.

Nor should the estimated employment adjustments generated by the agreements prove burdensome for any state. The largest percentage gain among the states is nine hundredths of 1 percent, estimated for Delaware, while the largest loss is 22 hundredths of 1 percent, for Mississippi (Table 2). More detailed analysis indicates similarly diminutive changes in employment for manufacturing sectors within the states. For no state does the estimated change in any sector exceed three-tenths of 1 percent of the state's total employment.

Table 1
Long-Term Employment Changes Estimated for U.S. Manufacturing Sectors as Result of Uruguay Round Agreements

SIC Code	Description	Percent Change in Employment
21	Tobacco Products	2.50
36	Electronic and Other Electrical Equipment and Components, except Computer Equipment	2.06
28	Chemicals and Allied Products	1.21
38	Measuring, Analyzing, and Controlling Instruments; Photographic, Medical and Optical Goods; Watches and Clocks	.88
37	Transportation Equipment	.77
35	Industrial and Commercial Machinery and Computer Equipment	.55
26/7	Paper and Allied Products; Printing, Publishing, and Allied Industries	.50
29	Petroleum Refining and Related Industries	.50
30	Rubber and Miscellaneous Plastics Products	.50
32	Stone, Clay, Glass, and Concrete Products	.50
20	Food and Kindred Products	.43
33	Primary Metal Industries	.07
22	Textile Mill Products	−.50
24	Lumber and Wood Products, except Furniture	−.50
25	Furniture and Fixtures	−.50
31	Leather and Leather Products	−.50
34	Fabricated Metal Products, except Machinery and Transportation Equipment	−.50
23	Apparel and Other Finished Products Made from Fabrics and Similar Materials	−10.00

Source: U.S. International Trade Commission, *Potential Impact on the U.S. Economy and Industries of the GATT Uruguay Round Agreements* (Washington, D.C.: June 1994); and author's computations.

III. U.S. Comparative Advantages and the Liberalization of Trade in Goods

The foregoing analysis suggests that the Uruguay Round liberalizations will not pose major adjustment problems for U.S. labor in manufacturing industries. But how do the estimated employment changes correspond with the competitive—or comparative—advantages of the United States? Are employment gains significant in sectors where the nation has a marked comparative advantage in international trade? A contrary finding would not by itself prove that the Uruguay Round will make the country worse off, but it would raise some doubts.

To illustrate the point, suppose that trade liberalizations were agreed only for goods that a country did not export. Foreign barriers limiting the demand for the country's exports were retained, while the coun-

[3] Note that the employment impact can differ appreciably from the income effect, which, as the introductory section indicated, is widely expected to be positive.

Table 2
Estimated Long-Term Employment Changes from Uruguay Round Agreements, by State

Area	Percent Change in Employment
United States	−.01
Delaware	.09
Indiana	.08
Connecticut, New Hampshire	.07
Illinois, Minnesota, Ohio	.06
Colorado, Michigan, Nebraska, Wisconsin	.05
Arizona, Kansas, South Dakota	.04
Idaho, Iowa, Massachusetts, Rhode Island, Vermont	.03
Oregon, Washington	.02
Alaska, District of Columbia, Maryland	.01
Maine, Montana, Nevada, New Jersey, New Mexico, North Dakota, Oklahoma, Utah, West Virginia, Wyoming	0
Florida, Louisiana, Missouri	−.01
California, Texas, Virginia	−.02
Arkansas, Hawaii, Pennsylvania	−.03
New York	−.05
Kentucky	−.08
Georgia	−.11
North Carolina	−.13
South Carolina, Tennessee	−.14
Alabama	−.21
Mississippi	−.22

Source: U.S. Bureau of Economic Analysis (for underlying employment data); U.S. International Trade Commission, *Potential Impact on the U.S. Economy and Industries of the GATT Uruguay Round Agreements* (Washington, D.C.: June 1994); and author's computations.

try's own barriers to foreign goods were lowered, allowing its effective demand for them to increase. The country's increased demand for foreign goods might cause their price to rise relative to the price the country received for its exports. This worsening of the terms on which the country traded could well reduce its economic welfare.

Partly for this reason, trade representatives typically bargain vigorously during negotiations such as the Uruguay Round in order to secure reductions in foreign barriers that will fully compensate for any reductions they are offering in their own countries' barriers. They seek liberalizations in sectors where their nations possess comparative advantages.

In the following analysis, revealed comparative advantage is measured, as is customary, by the ratio of the nation's (or state's) net exports (exports minus imports) in each commodity category to the sum of the nation's (or state's) exports and imports in that category.[4] This ratio, or index, can take any value between −1 and 1. The larger the algebraic value for a category *relative to the values for other categories*, the greater the country's revealed comparative advantage (or the smaller its disadvantage) in that category. The magnitude of the ratio for a category has little significance in and of itself, apart from comparison with the ratio magnitudes for other categories. Of course, protectionist barriers somewhat distort the ratios, but not crucially.

As reported in Table 3, a ranking of the commodity categories, or sectors, according to these ratios indicates that tobacco products and chemicals and allied products are the manufacturing sectors in which the United States has the greatest comparative advantage, while leather and leather products and apparel and related products are the sectors of greatest comparative disadvantage.[5] These rankings can and do change with underlying economic conditions, but marked changes are unlikely in the short run. Because the most recent and reliable export data for the states are for the period 1990–91, the ratios relate to those years. Moreover, the data available to the negotiators as they firmed up their bargaining positions probably were not much more recent.

Negotiators may bargain more strenuously on behalf of sectors that possess a comparative advantage if those sectors also exhibit rapid growth. Rapid growth and comparative advantage are closely associated in the United States, at least for the period 1987–92, as can be seen in Figure 2. The simple coefficient of correlation between revealed comparative advantage and percent change in value added for the manufacturing sectors is 0.81 and is highly significant statistically.

Instructive as the ratios in Table 3 are, they convey no information about the importance of the various sectors in the national economy. A related

[4] Algebraically, the formula is

$$\frac{X_i - M_i}{X_i + M_i},$$

where X_i and M_i represent exports and imports of the ith commodity. Because data on imports are not available by state, imports were computed on the assumption that each sector within each state experienced the same intensity of import competition—that is, the same ratio of imports to shipments—as the corresponding sector within the nation. Shipments data were taken from U.S. Bureau of the Census, *Exports from Manufacturing Establishments: 1990 and 1991* (Washington, D.C.: 1994).

[5] Tobacco's comparative advantage ratio probably is artificially raised by government support.

Table 3

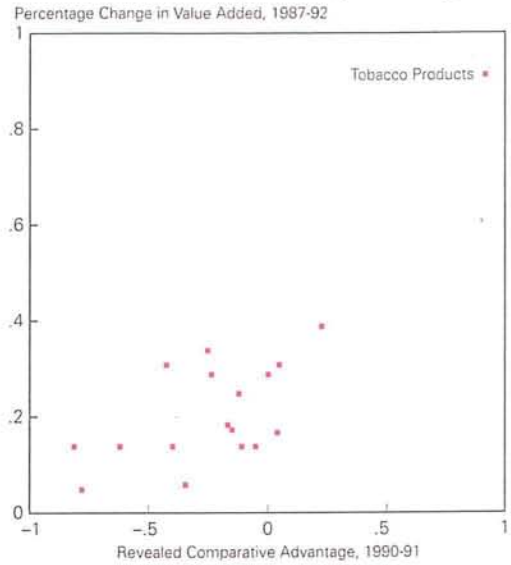
U.S. Revealed Comparative Advantage, by Manufacturing Sector, 1990–1991

SIC Code	Description	Revealed Comparative Advantage Ratio
21	Tobacco Products	.942
28	Chemicals and Allied Products	.252
20	Food and Kindred Products	.074
24	Lumber and Wood Products, except Furniture	.066
38	Measuring, Analyzing, and Controlling Instruments; Photographic, Medical and Optical Goods; Watches and Clocks	.028
35	Industrial and Commercial Machinery and Computer Equipment	−.025
34	Fabricated Metal Products, except Machinery and Transportation Equipment	−.083
26/7	Paper and Allied Products; Printing, Publishing, and Allied Industries	−.095
22	Textile Mill Products	−.124
37	Transportation Equipment	−.130
36	Electronic and Other Electrical Equipment and Components, except Computer Equipment	−.209
30	Rubber and Miscellaneous Plastics Products	−.225
32	Stone, Clay, Glass, and Concrete Products	−.318
33	Primary Metal Industries	−.371
29	Petroleum Refining and Related Industries	−.398
25	Furniture and Fixtures	−.593
31	Leather and Leather Products	−.753
23	Apparel and Other Finished Products Made from Fabrics and Similar Materials	−.786

Source: Export data from U.S. Bureau of the Census, *Exports from Manufacturing Establishments: 1990 and 1991* (Washington, D.C.: 1994). Import data from U.S. Department of Commerce (downloaded from COMPRO April 10, 1995).

Figure 2

Percentage Change in Value Added and Revealed Comparative Advantage for the United States, by Manufacturing Sector



Source: U.S. Bureau of the Census, *Exports from Manufacturing Establishments: 1990 and 1991* (Washington, D.C.: 1994); U.S. Department of Commerce (data downloaded from COMPRO April 10, 1995); staff, U.S. Bureau of the Census; and the author's computations.

consideration is that, other things equal, trade negotiators are likely to bargain more vigorously on behalf of the larger sectors. One way of taking these considerations into account is to weight the ratio for each sector by the sector's share of total manufacturing employment, so that the ratio reflects the sector's importance in manufacturing employment as well as its comparative advantage.

The ranking of sectors by these weighted ratios, in Table 4, differs appreciably from the ranking in Table 3. Chemicals and allied products and food and kindred products now occupy first and second places, with tobacco products fourth. A number of other noteworthy changes occur, although apparel and related products remains at the bottom of the list.

The question raised in the opening paragraph of this section can now be addressed with the aid of Tables 1 and 4. Rephrased to draw on those tables, the question becomes how the employment changes for the manufacturing sectors listed in Table 1 compare with the weighted ratios for those sectors in Table 4. Are the percentage employment gains from the negotiated liberalizations expected to be greater, or losses smaller, as weighted comparative advantage ratios rise in algebraic value? In particular, are substantial percentage employment gains estimated for sectors whose comparative advantage ratios are relatively high?[6] If so, the agreed liberalizations would seem

[6] Some analysts would prefer to compare the comparative advantage ratios with percentage changes in output rather than

Table 4

U.S. Revealed Comparative Advantage Weighted by Employment, by Manufacturing Sector, 1990–1991

SIC Code	Description	Revealed Comparative Advantage, Weighted by Employment
28	Chemicals and Allied Products	.013
20	Food and Kindred Products	.007
24	Lumber and Wood Products, except Furniture	.003
21	Tobacco Products	.002
38	Measuring, Analyzing, and Controlling Instruments; Photographic, Medical and Optical Goods; Watches and Clocks	.002
29	Petroleum Refining and Related Industries	−.003
35	Industrial and Commercial Machinery and Computer Equipment	−.003
22	Textile Mill Products	−.005
31	Leather and Leather Products	−.005
34	Fabricated Metal Products, except Machinery and Transportation Equipment	−.007
32	Stone, Clay, Glass, and Concrete Products	−.009
30	Rubber and Miscellaneous Plastics Products	−.011
26/7	Paper and Allied Products; Printing, Publishing, and Allied Industries	−.012
37	Transportation Equipment	−.013
33	Primary Metal Industries	−.015
25	Furniture and Fixtures	−.017
36	Electronic and Other Electrical Equipment and Components, except Computer Equipment	−.018
23	Apparel and Other Finished Products Made from Fabrics and Similar Materials	−.046

Source: Export and employment data from U.S. Bureau of the Census, *Exports from Manufacturing Establishments: 1990 and 1991* (Washington, D.C.: 1994). Import data from U.S. Department of Commerce (downloaded from COMPRO April 10, 1995).

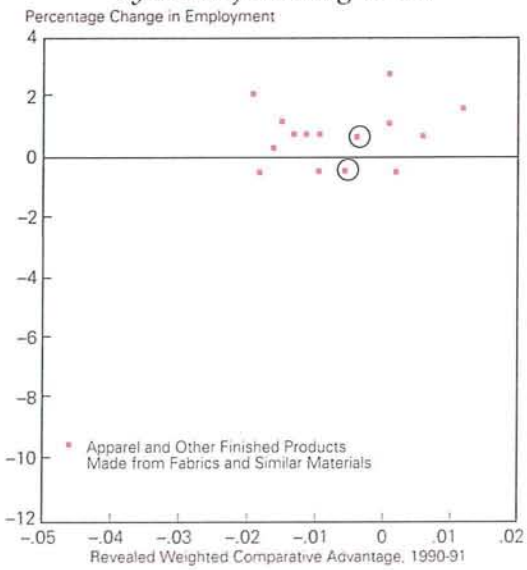
to allow the nation to capitalize on its comparative advantages.

As can be seen in Figure 3, the answer depends on the treatment given to the relatively extreme data for apparel and related products. If the data for that sector are included, the conclusion can be drawn that percentage employment gains do tend to rise with comparative advantage ratios; the simple coefficient of correlation between the two variables is 0.75, statistically significant at the 0.01 level. Such a conclusion fails to convey the pattern set by the data for the other 17 sectors, however. For them, the pattern is fairly random, and no statistically significant correlation exists between the percentage changes in employment estimated for them and their weighted comparative advantage ratios.

A similarly random pattern holds for each of the 50 states. As reported in Table 5, for 14 of the states a statistically significant positive correlation does exist between estimated percentage changes in employ-

Figure 3

Long-Term Percentage Change in Employment Due to the Uruguay Round and Revealed Comparative Advantage Weighted by Employment for the United States, by Manufacturing Sector



Note: Each of the circled points represents two observations.

Source: U.S. Bureau of the Census, *Exports from Manufacturing Establishments: 1990 and 1991* (Washington, D.C.: 1994); U.S. Department of Commerce (data downloaded from COMPRO April 10, 1995); U.S. International Trade Commission, *Potential Impact on the U.S. Economy and Industries of the GATT Uruguay Round Agreements* (Washington, D.C.; June 1994); and the author's computations.

percentage changes in employment. Thus, it should be noted that the Commission's estimates for percentage changes in output rarely differ from those for employment and that the few differences are minor.

Table 5
Coefficients of Correlation between Weighted Revealed Comparative Advantage and Estimated Long-Term Percentage Changes in Employment from Uruguay Round Agreements

Area	Number of Sectors	Correlation Coefficient	T-Statistic
United States	18	.75	4.56*
Alabama	15	.91	8.00*
Georgia	14	.88	6.28*
New York	15	.82	5.16*
South Carolina	14	.77	4.14*
Tennessee	16	.71	3.81*
Florida	15	.71	3.63*
Mississippi	13	.69	3.15*
Pennsylvania	17	.68	3.58*
Kentucky	14	.68	3.20*
West Virginia	10	.67	2.53*
New Jersey	16	.64	3.13*
California	15	.63	2.94*
North Carolina	17	.61	3.01*
Hawaii	3	.61	.77
Delaware	6	.58	1.43
Massachusetts	15	.56	2.45*
Virginia	13	.53	2.08
Idaho	7	.51	1.31
Texas	15	.43	1.74
Louisiana	12	.42	1.45
Wyoming	3	.28	.30
Arkansas	16	.25	.97
Rhode Island	10	.24	.69
Washington	13	.21	.71
Missouri	15	.19	.69
Colorado	11	.18	.55
Nevada	7	.16	.37
Maryland	13	.16	.53
Maine	10	.15	.42
Oregon	13	.13	.45
Utah	14	.11	.40
Oklahoma	15	.11	.39
Illinois	15	.10	.38
Connecticut	15	.10	.36
South Dakota	6	.07	.14
Ohio	15	.01	.05
Michigan	15	−.01	−.03
Minnesota	13	−.04	−.15
Indiana	13	−.05	−.15
Arizona	11	−.07	−.21
Wisconsin	16	−.09	−.33
Montana	4	−.09	−.13
Iowa	12	−.09	−.29
Alaska	3	−.13	−.13
Kansas	9	−.14	−.36
New Hampshire	12	−.17	−.55
Nebraska	13	−.18	−.61
Vermont	10	−.21	−.62
North Dakota	3	−.53	−.62
New Mexico	4	−.66	−1.23

*Significant at 0.05 level.

ment for the various manufacturing sectors and the weighted comparative advantages of those sectors; but in every case, just as for the nation, this outcome is attributable to extreme data for the apparel and related products sector. Once that "outlying" sector is excluded, no relationship seems to exist between expected percentage changes in employment and weighted comparative advantage in any of the states.[7]

This fairly random pattern suggests that foreign negotiators generally avoided granting relatively sizable trade liberalizations in sectors where the United States has a comparative advantage, and that U.S. negotiators generally avoided granting sizable liberalizations in sectors where the United States is at a comparative disadvantage. That such discretion should be exercised in the bargaining is not surprising, but both the United States and its trading partners might well have reaped greater gains in trade and income if the liberalizations had afforded more opportunity for both sides to capitalize on their comparative advantages.

By and large, both the United States and its trading partners apparently resisted granting sizable trade liberalizations in sectors where the other possessed a marked comparative advantage.

Of course, our analysis might have yielded a different conclusion if data had been available to allow the inclusion of nonmanufacturing, as well as manufacturing, sectors, and if data had been available at a more disaggregated, or detailed, level. Also, the long-term percentage changes in employment estimated to result from the Uruguay Round agreements are necessarily rather speculative in nature. On the other hand, it is well known that relatively little liberalization was agreed for a number of nonmanufacturing sectors in which the United States has substantial comparative advantages, including financial services, basic telecommunications, and audiovisual services, and this fact lends additional support to our conclusion.

[7] Note, however, that for a number of states data were available for only a few sectors, sharply limiting the number of observations on which to base this conclusion.

IV. Conclusion

Even allowing for a wide margin of error, our analysis suggests that the Uruguay Round agreements will have only a negligible impact upon employment in nearly every U.S. manufacturing sector, in every state, and in the country as a whole. Thus, in general, U.S. manufacturing industries and their employees will probably be spared from difficult adjustments.

This finding accords with another: The agreed trade liberalizations (as represented by the sectoral employment changes likely to result) seem to bear little relationship to the nation's revealed comparative advantages (weighted by employment). By and large, both the United States and its trading partners apparently resisted granting sizable trade liberalizations in sectors where the other possessed a marked comparative advantage. If so, both parties will be impeded from further specializing in the sectors of their greatest comparative advantage, and while world income will be stimulated by the agreements, it will grow by less than if both parties had been more forthcoming.

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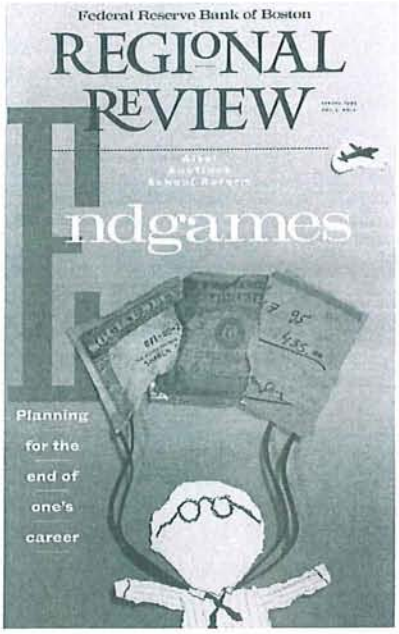

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An Assessment of the "Official" Economic Forecasts

Stephen K. McNees

Vice President and Economist, Federal Reserve Bank of Boston. *The author is most grateful to Lauren K. Fine and to Delia R. Sawhney for valuable research assistance.*

While the private sector generates dozens of macroeconomic forecasts, only three "official" forecasts emanate from government agencies. The oldest derives from the Employment Act of 1946, which created the Council of Economic Advisers and requires the Council to submit an annual report to the U.S. Congress. Starting in the early 1960s, this report provided an explicit numerical forecast of current-dollar or nominal GNP growth for the coming year; initially, the breakdown between the rate of real GNP growth and the rate of inflation, as measured by the implicit GNP deflator, had to be inferred from the text of the report. The Congressional Budget Act of 1974 created the Congressional Budget Office and requires it to present periodic reports on fiscal policy to the congressional budget committees. Significantly, this Act also established the practice of developing and presenting the federal budget, based on an economic projection, over a five-year horizon. Finally, in compliance with the Full Employment and Balanced Growth Act of 1978 (often referred to as the "Humphrey–Hawkins Act" in honor of its primary legislative sponsors), the Chairman of the Federal Reserve Board reports biannually to the Congress, presenting the economic projections of the members of the Federal Open Market Committee.

The Council of Economic Advisers' (CEA's) forecasts have been analyzed and compared with private forecasts many times. (See, for example, Moore 1977 and 1983 and McNees 1977 and 1988.) The Congressional Budget Office (CBO) has analyzed its own forecasts periodically, most recently in Reischauer (1995). This article updates these previous studies of CEA and CBO forecasts through 1994 and presents what may be the first published analysis of the Federal Open Market Committee's (FOMC's) "Humphrey–Hawkins" forecasts. It also incorporates alternative measures of changes in real output and prices recently developed by the Bureau of Economic Analysis (described in Young 1992 and 1993).

The results of the broader and longer data set used in this study alter slightly some of the conclusions of the previous research:

(1) Previous research almost uniformly has shown that the one-year-ahead "official" forecasts are about as accurate as forecasts obtained from surveys of private sector forecasters.[1] This study suggests the CEA's two- and four-year-ahead forecasts of real GNP have been slightly less accurate than, and the CBO's forecasts about as accurate as, the private sector forecasts. The lower accuracy stems from an optimistic bias in expectations of long-term real growth. On the other hand, the FOMC's forecasts issued each July for the following year are shown to have been somewhat more accurate than a standard private-sector forecast.

There are ample reasons to be skeptical that these differences will persist in future forecasts, however. For example, most of the advantage of the FOMC's forecasts derives from a superior performance in the early 1980s; since that time, private forecasts have been about as accurate as the FOMC's forecasts.

(2) Previous research has found both nominal and real GNP forecasts more accurate than simple rules of thumb, but at least one earlier study suggested that the "official" inflation forecasts were "not much if any better than simple extrapolations of last year's rate" (Moore 1983, p. 447 and Zarnowitz 1992, p. 399). This study finds that both the "official" and private inflation forecasts have been clearly more accurate than forecasts based on simple extrapolative rules of thumb.

I. Measuring Forecast Accuracy

A fundamental issue in judging forecast accuracy is the choice of the set of actual data to compare to the predictions. This follows from the fact that the National Income and Product Accounts (NIPAs) are revised repeatedly as new information becomes available. If the objective of the evaluation is to understand contemporaneous behavior—such as the reaction in the financial markets or the economic decisions taken at the time—the first data released, here called the preliminary data, are clearly the appropriate set of actual data to be used to evaluate the forecast. Preliminary data, however, include only partial information. Revised data, although not available until later, contain more information and, therefore, provide a better estimate of what actually occurred. They are the more appropriate standard for economic policymakers, econometric modelers, and most nonfinancial decisionmakers to use to estimate what "really happened" in the economy, as opposed to what was initially thought to have happened. These revised data will receive the primary emphasis in what follows, and the results based on preliminary data will be mentioned only secondarily.

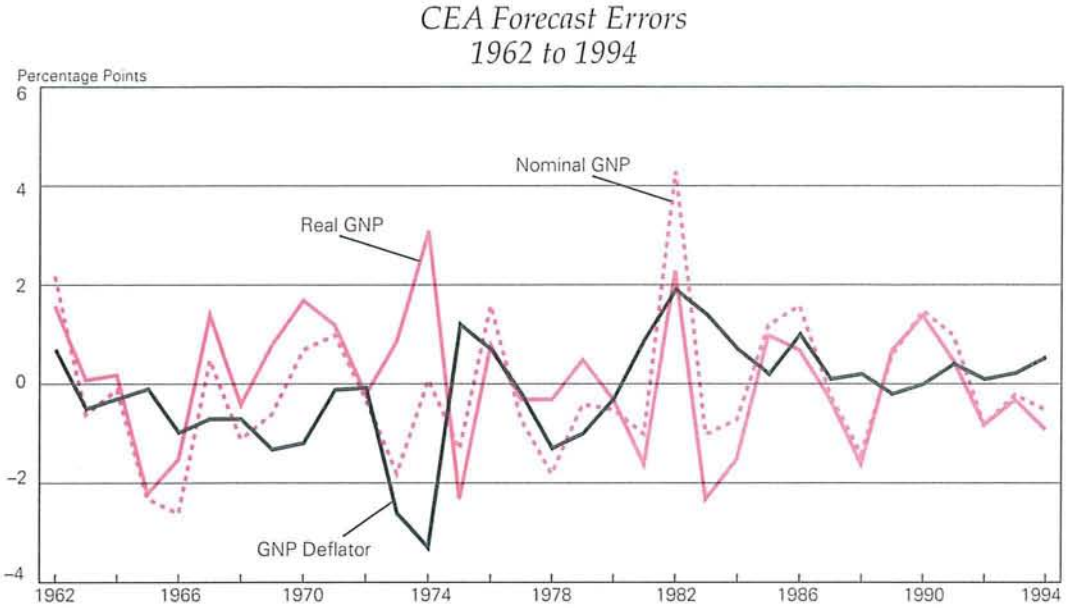
The CEA's two- and four-year-ahead forecasts of real GNP have been slightly less accurate than, and the CBO's forecasts about as accurate as, the private sector forecasts.

However, the fact that more recent data contain more statistical source information does not imply that forecasts should be evaluated against the actual data maintained in current data bases. The problem with doing so is that from time to time the BEA makes definitional and classification changes in the accounting framework (such as shifting mobile home production from consumer durable goods to residential investment) and also updates the base year of the weights by which the components of GNP are aggregated. Changes in base year weights affect the division of nominal GNP between prices and output.

Until recently, the BEA chose to emphasize GNP fixed-weighted quantity indexes because of their relative simplicity. The disadvantage of using a fixed-weighted quantity index, with weights chosen from one specific period, is that the relative price structure of the economy changes over time, clouding the interpretation of aggregate measures in periods far removed from the base year. This problem is especially acute for products like computers, whose relative prices have changed rapidly. The BEA has re-

[1] The exception (Belongia 1988) found that the CEA and the CBO forecasts were not biased but infers that "private sector forecasts generally were more accurate than those of the CBO" (p. 22) from a Fair-Shiller test that shows the private sector forecasts contained significant, independent information.

Figure 1



sponded to this issue by updating the base year from which the fixed weights are taken in its periodic benchmark revisions—in 1975, the base year was changed from 1958 to 1972; in 1985, the base year was changed from 1972 to 1982; and in 1991, the base year was changed from 1982 to 1987.[2]

It does not seem reasonable to hold economic forecasters responsible for anticipating such changes in the social accounting framework. For example, a forecast might predict every component of GNP perfectly in 1982 weights but be far off the mark for total GNP in constant 1987 dollars. Unfortunately, once the BEA changes its accounting framework or rebenchmarks the NIPAs to a more recent year, it stops issuing estimates of actual values on the old basis. This poses an insurmountable problem for the evaluation of multi-period forecasts; no set of actual data exists in the same accounting framework used by the forecasters for periods just after a benchmark revision. To evaluate the multi-period forecasts in 1982 dollars made just before the BEA's shift to 1987 dollar accounts, one must either adjust the forecasts to the new accounting framework or attempt to produce actual data in the previous system of accounts. Neither approach is very satisfactory.

Recently, the BEA introduced two alternative measures of real GNP which are not based on the price weights of a single base year. (See Young 1992.) This article utilizes these "benchmark-years-weighted quantity indices" to compute the actual values in those years for which no actual data are available in the same system of accounts in which the forecasts were made, specifically 1975, 1985, and 1991. The following sections assess the performance of CEA, CBO, and FOMC forecasts in turn, using this alternative measure where no comparable actual data exist.

II. The CEA Forecasts

Figure 1 displays the errors of the CEA's one-year-ahead forecasts of the growth rates of nominal GNP, real GNP, and the implicit GNP deflator issued each year since 1962. (See the appendix for a description of the data, and Moore 1983, Ch. 26, page 433 and especially Table 26-3 on pages 442 and 443 for data back to 1961.) The errors are highly variable: By far the largest errors for both real growth and inflation oc-

[2] In 1991, both the BEA and the forecasters shifted emphasis from GNP to GDP. Accordingly, the errors reported below simply splice the two concepts after 1991.

Table 1
CEA Summary Error Measures
1962 to 1994

	Nominal GNP	Real GNP	GNP Deflator
Mean Errors			
CEA	−.1	.1	−.1
"Naive 1"	−.3	−.4	−.1
"Naive 4"	−.3	−.3	−.2
Mean Absolute Errors			
CEA	1.1	1.1	.8
"Naive 1"	2.3	2.5	1.0
"Naive 4"	1.9	2.3	1.6
Root Mean Squared Errors			
CEA	1.4	1.3	1.1
"Naive 1"	2.8	3.1	1.5
"Naive 4"	2.3	2.8	2.0
	Percent of errors greater than:		
1 Percent	39	42	24
2 Percent	12	15	6

curred in 1974. Because those errors were of opposite sign and roughly equal in magnitude, however, the nominal GNP forecast for 1974 was quite accurate, as would be expected if the 1974 errors were attributable to an oil price "supply shock." By far the largest error in any of the variables during this period was the 4.3 percentage point overestimate of nominal GNP growth in 1982. Both real growth and inflation were overestimated by unusually large amounts, a particularly clear example of an unexpectedly large downward shift in aggregate demand.

The vast majority of forecast errors, however, are less than 1 percentage point. As shown in Table 1, the average error without regard to sign, or the mean absolute error (MAE), is 1.1 percentage points for nominal and real GNP and 0.8 percentage point for inflation. The square root of the mean squared error, the RMSE, ranges from a low of 1.1 percentage points for inflation to a high of 1.4 percentage points for nominal GNP. Because the variability of these three series, as measured by their standard deviations, is about the same in this period, about 2.5 percentage points, it is not unreasonable to say that inflation rate forecasts tended to be more accurate than the nominal or real GNP forecasts. It is sometimes alleged that "official" forecasts contain an optimistic bias; however, no evidence is seen here of such a bias, in that the average error of all forecasts for each variable, shown in the top panel, is essentially zero.

The question is often asked, "How good are these forecasts?" The answer depends on the alternative forecast to which they are compared: Relative to simple or even fairly complex rules of thumb, the CEA forecasts are clearly superior. Evidence for this proposition also appears in Table 1. The second row of each panel gives the summary error measures for a naive model that takes last year's growth as its forecast of this year's growth. The third row of each panel shows a naive rule of thumb in which the average rate of growth in the past four years is taken as the forecast of the next year. The CEA forecasts are substantially better than either of these simple formulas.[3]

The CEA's forecasts are about as accurate as the private sector forecasts made around the same time. An exact comparison of the two is difficult because it is not clear exactly when the CEA forecasts were finalized. The Administration's forecasting efforts start as early as November of the previous year in connection with the planning for the President's budget proposal. But it is clear that the forecasts could be modified just before the release of the Economic Report of the President, which often includes actual data for the fourth quarter of the prior year, not released until late in January. The CEA forecast tends to be slightly more accurate than private sector forecasts released in December or January and slightly less accurate than those released in February. Table 2 attempts to match the CEA forecast with the private sector forecast released at about the same time. Because the differences are so small and a comparable timing pattern difficult to establish, it appears that the accuracy of the CEA forecast and that of private sector one-year-ahead forecasts are essentially the same.

III. The CBO Forecasts

The major innovation associated with the institution of CBO forecasts is that, for federal budgetary planning purposes, they are required to cover a five-year horizon. This longer horizon opens the possibility that evaluations of short-term forecasts may differ from those of "long-term" forecasts. (See, for example, Kamlet, Mowery, and Su 1987.)

The CBO has issued a series of evaluations of the accuracy of its own forecasts relative to those of the

[3] The naive forecasts use the actual growth rates available at the time these hypothetical forecasts would have been made.

Table 2
A Comparison of Private and CEA Summary Error Measures
1972 to 1994

	Nominal GNP	Real GNP	GNP Deflator
Mean Errors			
ASA	−.2	−.1	−.1
DRI	−.5	−.3	−.2
CEA	.0	.1	−.1
Mean Absolute Errors			
ASA	1.2	1.0	.9
DRI	1.0	1.0	.8
CEA	1.1	1.1	.8
Root Mean Squared Errors			
ASA	1.6	1.2	1.3
DRI	1.3	1.3	1.1
CEA	1.4	1.3	1.2
	Percent of errors greater than:		
1 percent			
ASA	48	30	26
DRI	44	30	26
CEA	39	35	26
2 percent			
ASA	17	13	9
DRI	17	17	9
CEA	4	17	9

CEA and those from the private sector. (See, for example, Reischauer 1994 and 1995.) The evaluation in this article differs from the CBO's in two ways: (1) the CBO uses the benchmark-year-weighted actual data throughout, whereas this study uses that series only when no actual data exist in the same conceptual framework in which the forecast was made, as explained above; (2) the CBO uses the Blue Chip survey as its measure of private sector forecasts; because the horizon of the Blue Chip forecasts did not extend to two years prior to 1982, this limits the period of comparison. In this study the forecasts of a prominent private commercial forecasting organization are spliced onto the Blue Chip forecasts to create a time series reaching back to 1976, when the CBO started to forecast.

The results of the three-way comparison are summarized in Table 3. The top panel shows that the accuracy of two-year inflation forecasts is virtually identical for the CEA, the CBO, and the "private sector" standard. The middle panel shows that the CBO forecasts of real GNP growth over a two-year horizon were about the same as the private sector forecasts and slightly, but distinctly, more accurate than the CEA's forecasts. The bottom panel shows that the private sector forecasts of real GNP growth over a four-year horizon are slightly more accurate than the CBO's and noticeably more accurate than the CEA's long-term forecasts. Because all previous comparisons of the CEA's forecasts with private sector forecasts have suggested the two are about equally accurate, this result merits some investigation.

Note first that this is not an inconsistency; previous comparisons, including the ones described in Tables 1 and 2 above, cover a one-year horizon. The superiority emerging from the CBO's data set comes from forecasts of real GNP growth over the longer

Table 3
Comparison of Private and Public Forecast Errors
1976 to 1994

	CBO	CEA	"Private"[a]
A. Inflation, CPI—Two-Year Average Growth Rate			
Mean Error	−.1	−.2	.0
Mean Absolute Error	1.4	1.4	1.4
Root Mean Squared Error	1.8	1.8	1.8
	Percent of errors greater than:		
1 Percent	44	61	50
2 Percent	28	17	22
B. Real GNP—Two-Year Average Growth Rate			
Mean Error	.3	.4	.1
Mean Absolute Error	.8	1.1	.8
Root Mean Squared Error	1.0	1.3	1.0
	Percent of errors greater than:		
1 Percent	33	39	44
2 Percent	0	17	0
C. Real GNP—Four-Year Average Growth Rate			
Mean Error	.8	1.2	.5
Mean Absolute Error	.9	1.3	.8
Root Mean Squared Error	1.2	1.5	1.0
	Percent of errors greater than:		
1 Percent	31	44	38
2 Percent	19	19	6

[a]Forecasts of a prominent commercial forecasting organization, 1976 to 1981, and Blue Chip forecasts thereafter.

two- and four-year horizons. The source of the difference is fairly clear; the CBO and private sector forecast errors for two-year estimates never exceed 2 percentage points. In contrast, the CEA's real GNP forecast errors exceeded 2 percentage points for 1981–82, 1983–84, and 1990–91. The first and last were overestimates, the other an underestimate of the strength of the early recovery. In all three instances, the CBO and private sector forecasts were quite similar. Their errors were unusually large and of the same sign as the CEA's, but in no case did they exceed 2 percentage points. As for the four-year real growth forecasts, all three forecasters show some tendency toward optimism—all their mean errors were positive. The higher mean value for the CEA forecasts reflects the fact that all its errors were positive except for an underestimate of the strong 1983–86 recovery period.

The private sector forecasts of real GNP growth over a four-year horizon are slightly more accurate than the CBO's and noticeably more accurate than the CEA's long-term forecasts.

Private sector forecasts are often wrong. It is not uncommon to find the actual outcome either higher than the highest private sector individual's point forecast or lower than the lowest point forecast. These facts embolden many private sector forecasters, as perhaps they emboldened the CEA forecasters, to diverge sharply from the private sector consensus view. More often than not, however, this strategy fails. Although it is easy to say the private consensus forecast is wrong (because it usually is), it is generally extremely difficult to systematically guess the direction of its departure from the future reality.

IV. The FOMC Forecasts

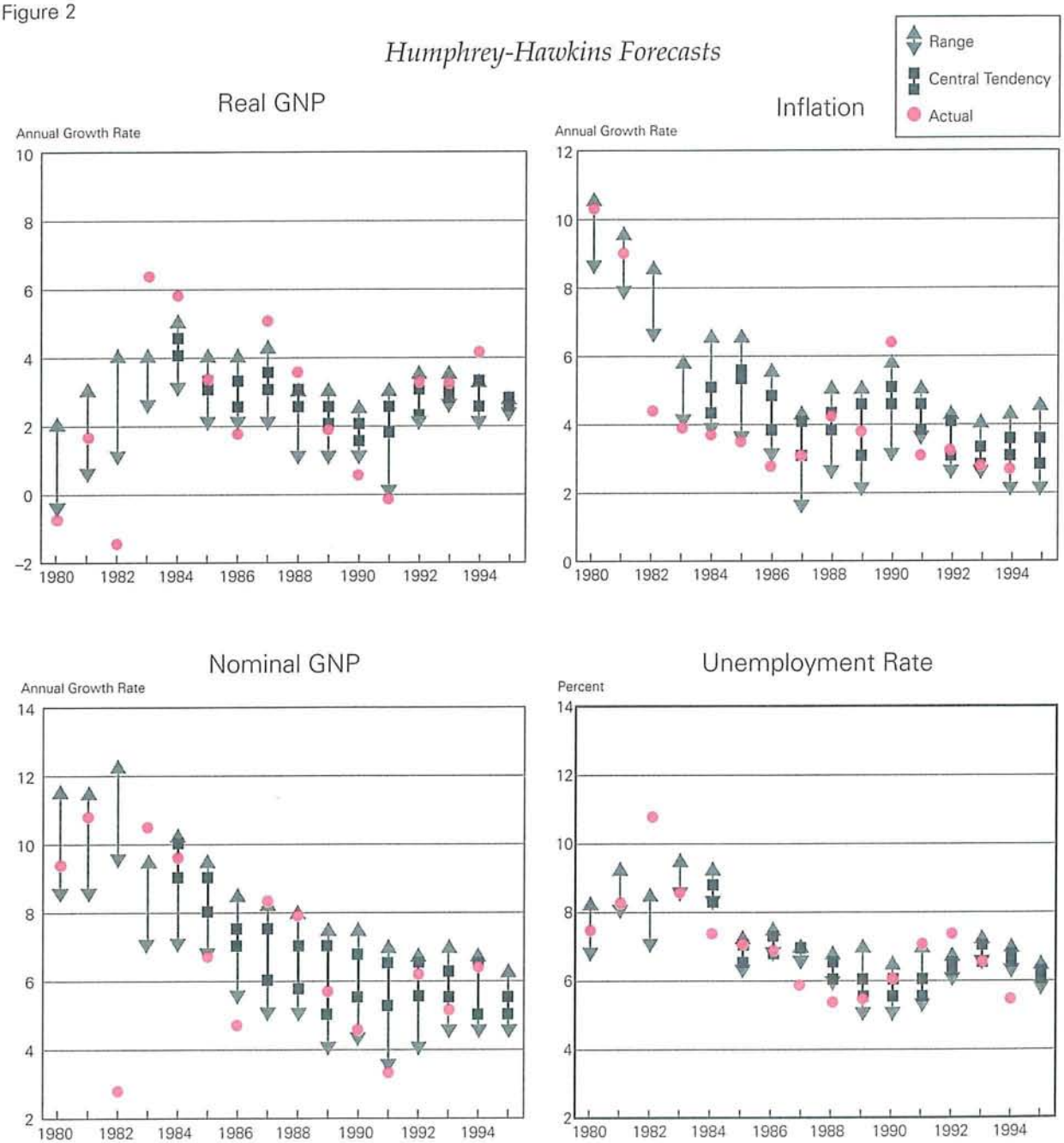
Since 1979, the Chairman of the Federal Reserve Board has appeared before Congress twice a year to discuss Federal Reserve conduct of monetary policy. In the testimony, the Chairman presents the range of the FOMC members' forecasts of the growth rates of nominal GNP, real GNP, and inflation (measured initially by the GNP deflator and more recently by the Consumer Price Index), and the level of the unemployment rate in the fourth quarter. In the February testimony, the forecasts pertain to the current year; in July, forecasts are given for both the current year and the following year. Currently, the range describes the forecasts of all participants at FOMC deliberations—that is, both voting and nonvoting FOMC members; earlier practice occasionally included only the forecasts of voting members.

Starting in 1983, the range of the forecasts was supplemented with a central tendency, constructed by discarding the extreme forecasts but not condensed to a point forecast. Because the FOMC forecasts are presented as a range, rather than as a point estimate as are virtually all other macroeconomic forecasts, assessing the FOMC forecasts cannot follow the standard procedures for point estimate forecasts.

For the one and one-half-years-ahead forecasts—those made each July for the following year, Figure 2 shows the high and the low points of the FOMC forecast range (the grey diamonds), the high and low points of the "central tendency" of the FOMC's forecasts (the black squares), and the actual outcome (the red circles) based on the revised data. Perhaps the most obvious way to start to assess these data is simply to compute the frequency with which the actual outcome fell within the range of FOMC members' forecasts, what will henceforth be called their "success rate." The success rate of the range of FOMC forecasts of all variables for all horizons relative to the preliminary data is 56 percent. The success rate relative to the revised data, as shown in the top panel of Table 4, is 49 percent. The difference between the two is due almost entirely to the real GNP forecasts—48 percent of the preliminary real growth estimates fell within the FOMC's range, whereas only 28 percent of the revised estimates of real growth fell within the FOMC forecast range, as shown in the second column, last row of the top panel of Table 4. In contrast, for the other three variables the success rates exceed 50 percent for both the preliminary and the revised actual data. The highest success rates are 67 percent for both the one-year-ahead inflation forecast and the one and one-half-years-ahead forecast of nominal GNP growth. The lowest success rate is the 19 percent rate for real GNP growth forecasts one-half year ahead.

Note that the success rates are consistently higher for the longer horizons. This generalization is true for all four variables, with preliminary or revised data,

Figure 2



Note: Forecast made in July for the following year, fourth quarter over fourth quarter.

Table 4
Success Rates
Percent of Revised Actuals within Forecast Interval

Forecast Horizon	Nominal GNP	Real GNP	Inflation Rate	Unemployment Rate	All Variables
		FOMC range, 1980 to 1994			
Half-year[a]	56	19	50	50	44
One Year	47	33	67	53	50
One and one-half years	67	33	53	60	53
All Horizons	57	28	57	54	49
		FOMC central tendency, 1983 to 1994			
Half-year	17	0	17	42	19
One Year	8	8	42	33	23
One and one-half years	36	0	36	36	27
All Horizons	20	3	31	37	23

[a]The half-year forecasts include a forecast made in July 1979 for 1979.

and for both the FOMC range and the FOMC central tendency, shown in the bottom panel of Table 4. The fact that forecasts with a longer horizon have a higher success rate is not at all a paradox: The success rate is the net result of two distinct factors—as the forecast horizon lengthens, forecast *uncertainty* rises, but at the same time the *dispersion* of the FOMC members' forecasts rises, as shown in Table 5. The dispersion among point forecasts is conceptually different from the uncertainty attached to each forecast. If the success rate were to be interpreted as a measure of uncertainty or a confidence interval, the level of confidence that can be associated with these forecasts is quite low—often below 50 percent. The decline in the success ratio as the forecast horizon lengthens simply shows that the dispersion of FOMC members' individual forecasts grows faster than the forecast uncertainty rises.

Table 5
The Width of the FOMC Ranges
1980 to 1994

Variable	Forecast Horizon	High	Mean	Low
Nominal GNP				
	Half-year[a]	3.0	1.91	1.25
	One Year	4.0	2.65	1.0
	One and one-half years	3.5	2.95	2.25[b]
Real GNP				
	Half-year[a]	2.5	1.41	.5
	One Year	3.0	1.90	1.0
	One and one-half years	3.0	2.00	1.0[c]
Inflation				
	Half-year[a]	1.75	1.16	.5
	One Year	2.0	1.58	.5
	One and one-half years	3.0	2.25	1.5
Unemployment Rate				
	Half-year[a]	1.25	.67	.25
	One Year	1.25	.80	.25
	One and one-half years	2.0	1.13	.5

[a]The half-year forecasts include a forecast made in July 1979 for 1979.
[b]Low falls to 1.75 for 1995.
[c]Low falls to 0.5 for 1995.

The dispersion of the FOMC members' point forecasts, as measured by the distance between the highest and the lowest forecasts, is not correlated with the accuracy of the forecasts, as measured by the distance between the actual outcome and the midpoint of the range of the FOMC forecasts. This same result occurs in virtually all collections of individual point forecasts, suggesting that the dispersion of individual forecasts is not a good measure of forecast uncertainty (McNees with Fine 1994).

Under the assumption that the FOMC forecasts can be characterized by their midpoint, one can easily compare them with other forecasts traditionally expressed as point estimates. Table 6 provides the standard summary error measures, the MAE and the RMSE, for the Blue Chip forecasts, the mean of a collection of private sector forecasts, and for the midpoint of the FOMC forecasts. Generally speaking, the

Table 6
Forecast Accuracy, Blue Chip and the Midpoint of the FOMC Range
1980 to 1994

	Mean Absolute Errors			
	Nominal GNP	Real GNP	Inflation	Unemployment Rate
Half-year[a]				
Blue Chip	1.4	1.3	.6	.5
FOMC	1.2	1.3	.7	.5
One year				
Blue Chip	1.6	1.3	.9	.5
FOMC	1.5	1.3	.8	.5
One and one-half years				
Blue Chip	1.9	1.6	1.4	.8
FOMC	1.7	1.4	**1.0**	.8

	Root Mean Squared Errors			
	Nominal GNP	Real GNP	Inflation	Unemployment Rate
Half-year[a]				
Blue Chip	1.8	1.5	.7	.7
FOMC	1.6	1.5	.9	.6
One year				
Blue Chip	2.3	1.6	1.2	.8
FOMC	2.1	1.5	1.1	.7
One and one-half years				
Blue Chip	3.0	2.1	1.7	1.2
FOMC	**2.5**	**1.7**	**1.3**	1.1

[a]The half-year forecasts include a forecast made in July 1979 for 1979.

two sets of forecasts are about equally accurate. The few instances where the summary error statistics differ by more than 0.2 percentage point are shown in bold type. All such cases occur for the longest horizon, one and one-half years—that is, the forecasts for the next year that the Federal Reserve Chairman presents in testimony each July. In each case, the FOMC's midpoint was more accurate than the Blue Chip consensus.

Table 7 sheds some light on the source of much of the superior performance of the FOMC midpoint forecasts. Table 7 compares the accuracy of the Blue Chip forecasts with the midpoints of both the range of the FOMC's forecasts and its central tendency.[4]

[4] Forecasts of a prominent commercial forecasting organization are used for the Blue Chip nominal GNP, inflation, and unemployment rate 6- and 18-month ahead forecasts made in July of 1979.

Table 7
Forecast Accuracy, Blue Chip and the Midpoints of the FOMC Range and Central Tendency
1983 to 1994

	Mean Absolute Errors			
	Nominal GNP	Real GNP	Inflation	Unemployment Rate
Half-year				
Blue Chip	.9	1.0	.6	.3
FOMC Range	1.0	1.0	.6	.3
FOMC CT	1.0	.9	.6	.3
One year				
Blue Chip	1.3	1.2	.8	.5
FOMC Range	1.3	1.2	.7	.4
FOMC CT	1.4	1.3	.6	.5
One and one-half years				
Blue Chip	1.3	1.0	1.2	.7
FOMC Range	1.3	1.2	**.9**	.7
FOMC CT	1.2	1.0	**.8**	.7

	Root Mean Squared Errors			
	Nominal GNP	Real GNP	Inflation	Unemployment Rate
Half-year				
Blue Chip	1.1	1.1	.7	.4
FOMC Range	1.1	1.1	.7	.4
FOMC CT	1.1	1.1	.7	.4
One year				
Blue Chip	1.4	1.3	.9	.6
FOMC Range	1.5	1.3	.9	.6
FOMC CT	1.5	1.4	.8	.6
One and one-half years				
Blue Chip	1.6	**1.2**	1.3	.9
FOMC Range	1.5	1.5	1.1	.8
FOMC CT	1.5	**1.2**	**1.0**	.8

Because the central tendency was not announced until 1982, this table omits the early 1980s when, as we have seen earlier, large forecast errors were made, especially for nominal GNP growth in 1982. (See the box.) With one exception, the table shows that the FOMC forecasts and the private forecasts are about equally accurate. The exception is the one and one-half-years-ahead inflation rate forecasts, where the FOMC forecasts are slightly but distinctly superior. Private forecasters might well argue that this advantage stems from the FOMC's ability to influence future inflation.

1982: A Forecast Failure.

Forecasts of 1982 were the least accurate, by a large margin, in the past 20 years. As illustrated below, all forecasters overestimated both real growth and inflation and thus overestimated nominal GNP growth by an extraordinary amount.

	Forecasts of 1982			
	Nominal GNP Growth	Real GNP Growth	Inflation Rate	Unemployment Rate
	Forecasts made in mid-1981			
Blue Chip	12.4	4.0	8.0	6.9
FOMC, midpoint of range	10.875	2.5	7.5	7.75
	Forecasts made in early 1982			
Blue Chip	9.9	2.6	7.2	8.7
FOMC, midpoint of range	9.25	1.75	7.125	8.875
Actual Values	2.7	−1.5	4.3	10.7

V. Summary

The historical record provides ample evidence that macroeconomic forecasts, both private and "official," are often imprecise and, at times such as 1973–74 and 1982, fundamentally misleading. There can be no assurance that such extraordinarily large surprises will not occur sometime again, and we have no good reason to be complacent about our ability to forecast.

At the same time, it is self-evident that the future is uncertain, so that forecasts necessarily will err. Perfection is not the relevant standard for judging forecast adequacy. When revisions often change actual outcomes by several tenths of a percentage point even well after the fact, it would be naive to expect forecast errors of essentially zero. From this perspective, it is comforting to see that multiple-percentage-point errors are rare. Far more often than not, macroeconomic forecasts have anticipated the level of the inflation and unemployment rates a year or more into the future within 1 percentage point. Simple rules of thumb have been far less reliable.

The only relevant standard for evaluating a forecast is the accuracy of other, comparable forecasts. If no superior alternative exists, then a large forecast error is simply a reflection of the fact that we live in an uncertain world, hardly a novel observation. On the other hand, because knowledge of the economy is quickly and widely disseminated, one seldom finds large, persistent differences in macroeconomic forecast accuracy among competent disinterested forecasters. This study reconfirms that both public and private forecasts are more accurate than simple rules of thumb. But it also finds some evidence of an optimistic bias in the multiyear real growth forecasts of the CEA and of slightly greater accuracy in the midpoint of the FOMC's longer-term forecasts than in their private sector counterparts. Much of this advantage stems from the turbulent early 1980s, however, and may not continue.

Appendix: Sources of Data

Figure 1: CEA Forecast Errors, 1962 to 1994 and
Table 1: CEA Summary Error Measures, 1962 to 1994

The forecasts of the Council of Economic Advisers come from the Office of Management and Budget, *Budget of the United States Government*, Fiscal Years 1963 to 1995. The CEA forecasts are made early each year for the current year. The actuals are the last available prior to each benchmark, taken from various issues of the *Survey of Current Business*. The years 1962 to 1964 are reported in 1954 dollars, 1966 to 1974 in 1958 dollars, 1976 to 1984 in 1972 dollars, 1986 to 1990 in 1982 dollars, and the remainder in 1987 dollars. The year-over-year forecasts made at the beginning of a benchmark year are in pre-benchmark dollars. The actuals, however, are released the following year in post-benchmark dollars. In all fairness to the forecasters' accuracy records, the benchmark-year-weighted index is used as the actual for 1965, 1975, 1985, and 1991.

Table 2: A Comparison of Private and CEA Summary Error Measures, 1972 to 1994

The dates of the private forecasts were consistently matched to the release date of the annual *Economic Report of the President*. See the paragraph above on Table 1 for information on the actuals.

Table 3: Comparison of Private and Public Forecast Errors, 1976 to 1994

The forecast errors for the two-year average growth rates of inflation and real GNP were calculated using forecasts and actuals provided by the Congressional Budget Office. For consistency, the benchmark-year-weighted index is used as the actual for 1985 and 1991. The remaining years use the last available actuals prior to each benchmark. The "private" forecasts were made by a prominent commercial forecasting organization, 1976 to 1981, and Blue Chip Economic Indicators thereafter.

Figure 2: Humphrey-Hawkins Forecasts
Table 4: Success Rates
Table 5: The Width of the FOMC Ranges, 1980 to 1994
Table 6: Forecast Accuracy, Blue Chip and the Midpoint of the FOMC Range, 1980 to 1994
Table 7: Forecast Accuracy, Blue Chip and the Midpoint of the FOMC Range and Central Tendency, 1983 to 1994

The forecast errors for the Humphrey-Hawkins and Blue Chip Economic Indicators forecasts were calculated using forecasts collected from the February and July Humphrey-Hawkins forecasts, the February and July issues of the *Blue Chip Economic Indicators*, and the last available actuals before each benchmark, as reported in various issues of the *Survey of Current Business*. The years 1979 to 1984 are reported in 1972 dollars, 1986 to 1990 in 1982 dollars, and the remainder in 1987 dollars. The 6- and 12-month-ahead forecasts made in February or July of a benchmark year are in pre-benchmark dollars. The actuals, however, are released the following year in post-benchmark dollars. In all fairness to the forecasters' accuracy records, the benchmark-year-weighted index is used as the actual for 1985 and 1991. To evaluate the 18-month-ahead forecasts made in July of the year prior to the benchmark, the benchmark-year-weighted index is also used for 1986 and 1992.

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Stocks, Bonds, Options, Futures, and Portfolio Insurance: A Rose by Any Other Name . . . .

Peter Fortune

Professor of Economics at Tufts University, now on leave at the Federal Reserve Bank of Boston. *The author is grateful to Lynn Browne, Richard Kopcke, and Katerina Simons for constructive criticisms of this paper.*

Trading volume and open interest in options and futures contracts on stock indices, equities, and interest rate instruments traded on world exchanges have experienced remarkable growth. From 1986 through 1991, the open interest in exchange-traded derivatives grew by 36 percent per year, reaching $3.5 billion at the end of 1991. The notional principal of financial derivatives traded in the even larger over-the-counter market (mostly on interest rates, in the form of swaps, forward agreements, and option-like caps, collars, and floors) grew at an annual rate of 40 percent.[1]

This rapid growth has been accompanied by controversy about the proper role of financial derivatives and the potential for abuse. Prominent attention has been given to losses by major corporations (for example, Procter & Gamble and Gibson Greetings on interest rate swaps), to the losses of broker-related short-term mutual funds (Piper Jaffray and Paine Webber on mortgage-backed securities), and to losses experienced by municipal agencies (Orange County, California, on just about everything).

Financial managers who believe they know the future course of interest rates and asset prices have always found ways to lose big, using traditional investments. Rightly, we have always put the responsibility for being "often wrong but never in doubt" on hubris, absolving the financial instruments themselves from responsibility. But now we are told by the media—a group notoriously ill-equipped to understand derivatives—that these "new things" are so complex that they are unknowable to all but a few. And those few are destined to misuse derivatives, it is claimed, because we do not understand what is going on and, not knowing how to ask the right questions, cannot limit the misuse. We now blame the instruments, not the arrogance of the owners or money managers. This counsel of despair leads the uninformed to believe that derivatives must be eliminated or regulated in order to prevent them from doing their damage.

Thus, the public debate about "derivatives" has promoted the impression that the heart of the problem has been a proliferation of brand new ways of making bets on future stock prices, interest rates, and exchange rates. The positive functions of derivatives as means of risk management are almost forgotten.

This article demonstrates how prices of *exchange-traded* stock index and equity options, as well as futures contracts, can be derived from information on an "option-replicating" portfolio of stocks and bonds that mimics the behavior of the option's premium. Using the equivalence between an option or futures contract and its replicating portfolio, the article demonstrates that exchange-traded options are really nothing new. Rather, they are repackages of the same traditional financial instruments. The article pursues this point by outlining several related risk-management strategies using options and futures contracts. These include dynamic hedging and its related strategy, portfolio insurance. Finally, the article addresses some circumstances in which "derivatives" are not equivalent to traditional instruments. These limitations are most common in the over-the-counter markets where custom-made derivatives are designed for specific uses.

I. The Pricing of Options and Futures

An equity option is a contract allowing the holder to buy or sell a fixed number of shares at a fixed price (the strike price) on or before an expiration date. The holder will exercise the option only if it is in his interest to do so. Thus, an equity option gives its holder the right, not the obligation, to buy or sell at a fixed price. The person who gives the option is called the *writer*, and for every option held an option must be written. The option is a call if the holder has the right to buy (take delivery of) the shares upon payment of the strike price; the writer must deliver the shares if the option is exercised. The option is a put if the holder has the right to sell (deliver) the shares upon receipt of the strike price; the writer of the put must take delivery if the put is exercised. The option is "European" if it can be exercised only on the expiration date, and "American" if it can be exercised at any time up to the expiration date. All equity options traded on U.S. exchanges are American-style options.

A stock index option is similar to an equity option with two important differences. First, the underlying security is a stock price index (for example, the S&P 500, the S&P 100), not a traded security. Secondly, the settlement is in cash rather than in securities. The owner of a call option on the S&P 500 will, upon choosing to exercise, receive the cash equivalent of the excess of the S&P 500 over the strike price rather than take delivery of the securities. All stock index options traded on U.S. exchanges are American-style with the exception of the S&P 500 index option.

An option's market price, or *premium*, is the sum of two components. The *intrinsic value* is the amount that will be received if the holder chooses to exercise the option immediately.[2] The intrinsic value of the option cannot be negative, for the holder would never choose to exercise the option if it reduces his wealth. Hence, the intrinsic value of a call option is denoted as $max(S - X, 0)$, where S is the current stock price and X is the strike price specified in the option contract. This notation simply means that the payoff is the larger of two values, $S - X$ or zero. The intrinsic value of a put option is $max(X - S, 0)$, which also cannot be negative, for the holder of a put will never choose to exercise it if the amount he receives (the strike price) is less than the current stock price.

The value of an option (its premium) will equal the intrinsic value only at the moment of expiration. Prior to expiration, the option will have a *time value*, which reflects the potential for the profitability of the option to change. Thus, an *out-of-the-money* call option, which has zero intrinsic value because the stock price (S) is less than the strike price (X), will still sell at a positive premium because investors realize that the option might become in-the-money at a later date, should the stock perform sufficiently well.

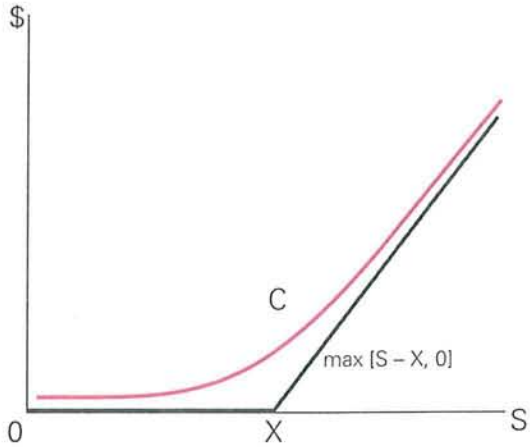
Figure 1 shows the typical relationship between the premium on a call option and its intrinsic value. The intrinsic value is the black line, which has a zero value when the stock price is at or below the strike price but increases dollar for dollar with the stock price when the call is in-the-money. The call premium, denoted by the red curved line labeled C, increases

[1] For a description of this growth, see Remolona (1993). Notional principal is the value of the contract upon which payments are based. It is considerably greater than the market value of the contracts with which it is associated. First, for most contracts the market price is well below the notional principal upon which payments are based. Second, notional principal involves double-counting. If, say, the holder of an interest rate swap for $1,000,000 (which has a net market value of zero) offsets it by selling a similar contract, the "true" net value is zero, but the reported notional principal will be $2,000,000.

[2] Immediate exercise of a call option will require the holder to pay the strike price (X) in exchange for shares valued at the market price (S). The profit is $S - X$ when S exceeds X. If S is less than X, the holder will not exercise the option and it will expire without value.

Figure 1

Call Premium and Intrinsic Value



from almost zero when the stock is without value and approaches the intrinsic value as the stock price gets very high. The vertical distance between the premium and the intrinsic value is the time value for that stock price. Time value is at its maximum for an at-the-money option.

The convex shape of the call premium–stock price relationship plays an important role in understanding option pricing. The intuition underlying this convexity is straightforward. If the stock price is very low, the option will be so out-of-the-money that it would take a rare boom in stock prices to become in-the-money; very little time value would be given to such an option. On the other hand, if the option is very deep in-the-money, it would take a rare downdraft to put it out-of-the-money, and the value of the option will be close to the intrinsic value.

The Pricing of European-Style Options

This section lays out the logic of option pricing in its simplest form. It relies on a standard assumption of economics, the "no free lunch" assumption: Riskless arbitrage opportunities arise only in disequilibrium and will not exist when security prices are in equilibrium. This means that if a portfolio of stocks and bonds can be constructed to match movements in the option premium, profit-seeking traders will ensure that, when markets are in equilibrium, no profitable arbitrage between the option and its *replicating portfolio* can occur. This assumption allows the option premium to be inferred from the value of its replicating portfolio.

Assume that the option in question is a European equity option on a stock whose daily price movements are binomial; that is, if the current stock price is S, the price on the next day will be either μS if the price goes up or δS if it goes down. Thus, μ is one plus the rate of increase and δ is one plus the rate of decrease.

The analysis of a European option can be summarized in a binomial tree. Consider a simple two-period call option, one that has a premium of C dollars in the first period and expires in the second period; the value of the premium is to be determined. Because the underlying stock price will increase to μS or decrease to δS, the value of the option at expiration on the second day will be either $max(\mu S - X, 0)$ or $max(\delta S - X, 0)$. To be specific, assume an option with a strike price of \$48 on a stock with a price of \$50 ($X = 48$, $S = 50$). If the stock price will either increase or decrease by 5 percent ($\mu = 1.05$ and $\delta = 0.95$), the next-day stock price will be either \$52.50 or \$47.50, and the payoff of the option will be either \$4.50 or zero.

If the option premium and stock price after an "up" day and a "down" day are denoted C_u and C_d, and S_u and S_d, respectively, the binomial tree for a two-period option is

$$C \begin{cases} C_u = max[S_u - X, 0], & \text{where } S_u = \mu S \\ C_d = max[S_d - X, 0], & \text{where } S_d = \delta S \end{cases} \tag{1}$$

The market price of the option on the first day cannot be determined without further information. That information is provided by noting that exactly the same final values could be achieved by investing in a portfolio of stocks and bonds; this is the option-replicating portfolio. Because the option-replicating portfolio is designed to have exactly the same payoff structure as the option, and because we know the final payoff structure of the option, the option must have exactly the same value as the option-replicating portfolio. The reason is that smart money knows that two assets worth exactly the same at any future time must be worth the same in the present, if arbitrage opportunities are to be eliminated. (The assumption that economic agents will act to eliminate arbitrage profits is a crucial foundation of finance theory.)

Suppose that the option-replicating portfolio consists of Δ ("delta") shares of the stock plus an investment of $\$B$ in bonds. Thus, a portfolio is simply a

choice of the values of Δ and B. If bonds pay $\$rB$ on the following day (r is 1.0 plus the riskless interest rate), the binomial tree for the option-replicating portfolio is

$$(\Delta S + B) \begin{cases} (\Delta S_u + Br), & \text{where} \quad S_u = \mu S \\ (\Delta S_d + Br), & \text{where} \quad S_d = \delta S \end{cases} \tag{2}$$

The next step in determining the call premium is to find the values of Δ and B that represent a portfolio of stocks and bonds with final values exactly matching the final values of the call option. That means that the option-replicating portfolio must satisfy the two equations describing the end-points of (1) and (2): $(\Delta S_u + Br) = C_u$ and $(\Delta S_d + Br) = C_d$, where C_u and C_d are known from the option's characteristics and S_u and S_d are known from the assumed values of S, μ, and δ. The required values of Δ and B are

$$\begin{aligned} &\Delta = (C_u - C_d)/(S_u - S_d), \\ &B = (C_u - \Delta S_u)/r, \quad \text{and} \\ &C = \Delta S + B, \\ &\text{where} \quad S_u = \mu S \quad \text{and} \quad S_d = \delta S. \end{aligned} \tag{3}$$

Suppose, as before, that a call option in question has a strike price of \$48 and that the current stock price is \$50, putting the option in-the-money with an intrinsic value of \$2. Assuming a 5 percent increase or decrease, the stock price will go to either \$52.50 or \$47.50 the next day. Under these assumptions, we have seen that the payoffs for this option must be either \$4.50 if the stock goes up or zero if it goes down. If the riskless interest rate is 1 percent ($r = 1.01$), the option-replicating portfolio will have a delta of 0.90 and the option-replicating portfolio will be a leveraged purchase of \$45 of stock financed by \$42.33 of debt, with a net value of \$2.67.

The final step requires another "no free lunch" assumption. If any two securities are known to have the same values at any future point in time, they must, in equilibrium, have the same values at every point in time. If they did not, traders would find profitable arbitrage opportunities and their actions would eliminate those opportunities, forcing prices into conformity. For example, we have seen that the option and its replicating portfolio are both worth either \$4.50 if the day is "up" or zero if it is down, and that the option-replicating portfolio is worth \$2.67 at the outset. Suppose that the call premium is only \$2. In this case, traders would buy the call and short the replicating portfolio, receiving a net amount of 67 cents. Because the final values of the option and the portfolio are equal, they will lose nothing at the end of the first period—increases in one are matched by declines in the other. Thus, they make a net profit of 67 cents with no risk. Traders would take advantage of this opportunity unless the call premium rose to \$2.67, exactly matching the value of the replicating portfolio. If, on the other hand, the initial value of the call had been \$3, traders would have sold the call and bought the option-replicating portfolio for \$2.67. They would make 33 cents with absolutely no risk, because at the end of the first period any profit or loss on the call is offset by loss or profit on the replicating portfolio.

This three-step analysis shows that, in an equilibrium with no arbitrage profits, the call premium at the outset must be equal to the value of the option-replicating portfolio. This simple example illustrates a key point of this article: A European call option is precisely equivalent to a portfolio of traditional securities, specifically, to a leveraged purchase of the underlying security. All that can be done with one can also be done with the other. Thus, caution must be used when interpreting statements that attribute some special qualities to options. For example, when options are described as allowing "high leverage," we should see that they have no special advantage in providing leverage; they are an alternative way of achieving a leveraged position, and in equilibrium they should cost about the same as the traditional way.

The full derivation of a binomial option pricing model for multiple time periods is given in Box 1. This involves solving the full binomial tree *backward* in the manner just outlined: From any two adjacent final payoffs, the value of the option at the preceding node can be computed, allowing derivation of the full range of option values on the day before expiration. Then, armed with those data, the option values at each node on the second day before expiration can be constructed. As developed in Box 1, at each node the value of the call option can be derived as

$$C = [qC_u + (1 - q)C_d]/r, \tag{4}$$

$$\text{where} \quad q = (r - \delta)/(\mu - \delta)$$

This recursion formula can be shown to be equivalent to the option-replicating formula $C = \Delta S + B$, but it puts the call premium into a recursive format that reveals the connection between current and future

Box 1: The General Binomial Option Pricing Model

The logic of the binomial pricing model is laid out in the text for a two-period option. Here we derive its general form for a multi-period European call option.

At time t a European call option with strike price X is written on an underlying stock with price S. The option expires at time T, at which time it will pay the holder $max[S_T - X, 0]$, the excess of the stock price over the strike price, if positive, or zero.[3] The stock's price follows a Bernoulli process: On each day it either increases to μ ($\mu > 1$) times the previous day's price with probability π, or falls to δ ($0 < \delta < 1$) times the previous day's price with probability $1 - \pi$. Thus, μ is 1 plus the rate of increase and δ is 1 plus the rate of decrease. The statistical expected value and variance of the one-period rate of return are $\pi(\mu - 1) + (1 - \pi)(\delta - 1)$ and $\pi(1 - \pi)(\mu - \delta)^2$, respectively.

We first consider the final payoffs at expiration. An option with $T - t$ periods will have $T - t + 1$ payoffs. If x is the number of "up" days in the remaining $T - t$ days, the payoff will be $max(\mu^x\delta^{T-t-x}S - X, 0)$. If there are too few "up" days, the payoff will be zero because the option will expire out-of-the-money. We can derive the critical number of good days, defined as the minimum number of "up" days required to put the option just at-the-money. This occurs when $x < x^*$, where $x^* = ln(X/S\delta^T)ln(\mu/\delta)$ is the minimum number of "ups" required to put the option at-the-money.

Consider any two adjacent final payoffs valued at $C(x, T) - max(\mu^x\delta^{T-x}S - X, 0)$ and $C(x + 1, T) = max(\mu^{x+1}\delta^{T-(x+1)}S - X, 0)$. These differ only because of the presence or absence of an "up" on day $T - 1$. Because the call can be hedged by an option-replicating portfolio, it can be shown that that arbitrage ensures that the option's value on the previous day is

$$C(x, T - 1) = r^{-1}[qC(x + 1, T) + (1 - q)C(x, T)] \quad \text{(B1.1)}$$

where q is the risk-neutral probability of an "up." This is the fundamental recursion formula described in the text. The analysis of the hedged position reveals that the probability parameter is $q = (r - \delta)(\mu - \delta)$, which does not depend upon the statistical probability (π) or on the expected return on stocks.

This recursive equation can be used to solve the whole binomial tree back to the beginning. Thus, starting with final values $C(x, T)$ and $C(x + 1, T)$ we can solve for the previous node $C(x, T - 1)$. Doing the same for the next two adjacent final payoffs we can find $C(x - 1, T - 1)$, allowing us to find $C(x - 1, T - 2)$, and so on.

Suppose that we are on day t of the option's life. Defining $(T - t, i)$ as the number of ways that there can be i "ups" in the remaining $T - t$ days, we can see that the call premium at any day after x "ups" (and $n - x$ "downs") is

$$C_t = r^{-(T-t)} \sum_{i=0}^{T-t} q^i(1 - q)^{T-t-i} max(S\mu^i\delta^{T-t-i} - X, 0) \quad \text{(B1.2)}$$

This says that the call premium is the present value of the expected final payoffs, using risk-neutral analysis, that is, using the risk-neutral interest rate as the discount rate, and computing expectations using the "objective" risk-neutral probability of an "up." The probability distribution used is the binomial distribution, hence the name binomial option pricing.

option prices. It states that the call premium at any node is the present value of the expected call premium at the next node. The expected call premium is a weighted average of the known call premiums in the "up" and "down" states of the stock. The parameter "q," called the *risk-neutral probability* of a stock price increase, sets the weights given to the "up" and "down" states. For our example (with $\mu = 1.05$, $\delta = 0.95$ and $r = 1.01$), this probability is $q = 0.60$ and the option premium implied by equation (4) is \$2.67.

Somewhat paradoxically, the option is valued as if investors are risk-neutral, that is, the premium depends upon the expected present value defined by the risk-neutral probability and the riskless interest rate. It is not that investors are truly risk-neutral, but rather that in pricing options they can be treated as if

[3] An essential feature of the binomial model is that time is divided into discrete intervals. We call these "days," with no necessary connection to our circadian rhythms.

Table 1
Value Matrix for Ten-Period European Call Option
No Cash Dividends

	Number of Ups (x)										
Period	0	1	2	3	4	5	6	7	8	9	10
0	10.91										
1	6.77	14.27									
2	3.60	9.29	18.36								
3	1.48	5.26	12.51	23.22							
4	.36	2.34	7.54	16.52	28.85						
5	.00	.63	3.66	10.60	21.37	35.24					
6	.00	.00	1.10	5.63	14.55	27.01	42.39				
7	.00	.00	.00	1.94	8.49	19.43	33.41	50.32			
8	.00	.00	.00	.00	3.41	12.44	25.15	40.52	59.12		
9	.00	.00	.00	.00	.00	5.98	17.53	31.51	48.42	68.88	
10	.00	.00	.00	.00	.00	.00	10.50	23.21	38.58	57.18	79.69

Parameters: $\mu = 1.10$, $\delta = 0.9091$, $\rho = 1.02$, S = X = 50. Note that $q = 0.5809$.

they are. As a result, the option premium is independent of the *statistical* probability of a stock price increase, and of the *statistical* expected rate of return on the stock. Rather, the option is valued using the riskless rate of interest, not an interest rate containing market risk.

The disconnect between option prices and the expected returns on the underlying assets appears paradoxical, for how can the value of a call option not be higher when the expected rate of increase of the stock price is higher? The answer lies in the ability to create a riskless arbitrage by buying a call and selling its option-replicating portfolio. Smart money will realize that a call option combined with a short position in its option-replicating portfolio is a perfect hedge, creating a riskless position requiring no net investment. The option premium will not contain any reward for risk, for while the option is risky in isolation, it has a perfect hedge and holding the option carries no inherent risks. The investor who decides to hold an unhedged option must do so without any expectation of reward, for the risk he bears is a matter of individual choice and is not inherent in the option itself. In the language of portfolio theory, any risks borne by the option holder are idiosyncratic, not systematic, and can earn no reward.

An example of the multiperiod valuation model illustrated in Box 1 is given in Table 1, which assumes a 10-period at-the-money European call option on a \$50 stock, with $\mu = 1.10$, $\delta = 1/\mu = 0.9091$, and $r = 1.02$. Each cell, equivalent to a node on the binomial tree, shows the value of the call on the n*th* trading day after x "ups" and $10 - x$ "downs"; this is denoted as $C(x, n)$. To compute the option premium in each cell we begin at the end, with the possible payoffs at expiration on day 10. These possible payoffs are computed as $max(\mu^i\delta^{10-i}S - X, 0)$; hence, each differs because of the different numbers of "up" and "down" days over the 10-day lifetime of the option. We see that the expiration-day values are zero for five or fewer ups and rise to \$79.69 for 10 consecutive "ups." These intrinsic values must be the call premiums at expiration because no time remains to receive a time value.

The day-9 option premiums can then be constructed using the known day-10 payoffs along with equation (4), using $q = 0.5809$. For example, if on day 9 there have been eight "ups," then by day 10 there must have been either eight or nine "ups," with payoffs of \$38.58 or \$57.18, respectively. Following equation (4), the premium at the (8,9) node must be $C(8,9) = \$48.42$. Computing all the possible call premiums on day 9 allows the day-8 premiums to be computed, and so on. Tracing the values back to the beginning, we see that the initial premium on this call option will be \$10.91.

Tables 2 and 3 show the option-replicating portfolio for our hypothetical 10-day European call option whose values are shown in Table 1. Table 2 reports the value of delta (Δ) for our hypothetical 10-day call option, while Table 3 shows the investment in bonds, $B(x, n)$, required to replicate the call option; this

Table 2
Option-Replicating Number of Shares

	Number of Ups (x)									
Day	0	1	2	3	4	5	6	7	8	9
0	.79									
1	.65	.86								
2	.48	.76	.93							
3	.28	.60	.86	.97						
4	.10	.38	.73	.93	.99					
5	.00	.15	.52	.85	.98	1.00				
6	.00	.00	.25	.69	.95	1.00	1.00			
7	.00	.00	.00	.39	.86	1.00	1.00	1.00		
8	.00	.00	.00	.00	.63	1.00	1.00	1.00	1.00	
9	.00	.00	.00	.00	.00	1.00	1.00	1.00	1.00	1.00

Parameters: $\mu = 1.10$, $\delta = 0.9091$, $\rho = 1.02$, S = X = 50.
Note: The option-replicating number of shares is the number of shares that results in value changes that match the change in the value of one European call option.

Table 3
Option-Replicating Investment in Bonds per European Call

	Number of Ups (x)									
Day	0	1	2	3	4	5	6	7	8	9
0	−28.40									
1	−22.99	−33.28								
2	−16.18	−28.70	−37.73							
3	−8.92	−21.98	−34.54	−41.33						
4	−2.94	−13.53	−28.82	−39.85	−43.82					
5	.00	−5.16	−20.04	−36.15	−43.89	−45.29				
6	.00	.00	−9.06	−28.66	−42.79	−46.19	−46.19			
7	.00	.00	.00	−15.90	−38.84	−47.12	−47.12	−47.12		
8	.00	.00	.00	.00	−27.92	−48.06	−48.06	−48.06	−48.06	
9	.00	.00	.00	.00	.00	−49.02	−49.02	−49.02	−49.02	−49.02

Parameters: $\mu = 1.10$, $\delta = 0.9091$, $\rho = 1.02$, S(0) = 50, X = 50, N = 10.
Note: The replicating investment in bonds is defined as B(x, n) = C(x, n) − δ(x, n) * S(n). A negative value indicates borrowing.

depends on the number of "ups" and "downs."[4] From these tables we see that at the outset the call is equivalent to 0.79 shares plus borrowing of $28.40. However, if 3 "ups" have occurred by day 5, the option-replicating portfolio consists of $36.15 in debt plus 0.85 shares. In the lower left portion of the matrices, the option is so far out-of-the-money that no shares are bought and no debt is incurred. Thus, the option is worthless because it cannot end in-the-money. In the lower right portion, the option is so deep in-the-money that one share is required to replicate one option.

Pricing of Put Options

From the call option pricing model it is easy to construct a pricing model for a European put option by invoking the *put–call parity theorem*. According to this theorem, arbitrage enforces a simple relationship between put and call premiums. A put and a call for the same stock, each with the same strike price and expiration date, must be priced so that at any time t the following is satisfied (P is the put premium):

[4] The cells are computed as $\Delta(x, n) = [C(x + 1, n + 1) - C(x, n + 1)]/[(\mu - \delta)S(n)$ and $B(x, n) = [\mu C(x, n + 1) - \delta C(x + 1, n + 1)]/[(\mu - \delta)S(n)]$.

$$P_t + S_t = C_t + Xr^{-(T-t)} \qquad (5)$$

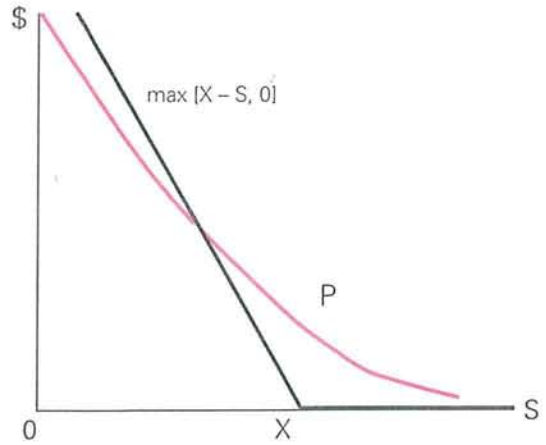
A simultaneous investment in a put and one share of the stock must be equal to an investment in a call plus bonds equal to the present value of the strike price. Arbitrage forces this to be true because the final values of the two positions are equal: At expiration on day T the stock *cum* put will be worth $S_T + max(X - S_T, 0)$, which is the greater of the exercise price or the stock price. On that same date the call *cum* bond position will be worth $max(S_{T-X}, 0) + X$, also equal to the larger of the stock price or the exercise price. Because two positions worth the same amount at one time must, in equilibrium, be worth the same at any other time, relationship (5) must hold.

From put–call parity we see that a put is equivalent to a call plus bonds equal to the present value of the strike price plus a *short* position in the stock. Once the equilibrium call premium is known, the equilibrium put premium is also known. Thus, in the case of *European* options, put pricing reduces to a simple transformation of call pricing. This is not true of American put options, for which there is no put–call parity relationship.

Figure 2 shows the typical relationship between the put premium and the stock price. The intrinsic value is shown by the black line and the put premium is shown by the red convex curve. There is one notable difference between the call and put relationships shown in Figures 1 and 2: For a call, the time value is always positive, but for a put it can be negative if the stock price is sufficiently low. That is, the premium described by put–call parity for a European put can be less than the intrinsic value if the put is deep-in-the-money. This anomaly of a negative time value for deep-in-the-money puts means that there can be an incentive to exercise the put early. For example, if the intrinsic value of the put is $10 and the put premium is at the put–call parity level of, say, $8, traders will want to buy the put at $8 and receive $10 by exercising it. This behavior is not possible for a European put, which cannot be exercised early. But it does present problems for an American put because the American put premium cannot go below its intrinsic value; if it did, traders would make a riskless profit by buying the put and immediately exercising it. Therefore, it is more accurate to say that the observed American put premium will be the higher of the intrinsic value or the put–call parity value.

Figure 2

Put Premium and Intrinsic Value



Thus, because no value is attached to the ability to exercise an American option early, the American call option must be priced as if it were a European call option, and the pricing model just outlined should work for both American and European call options. But an American put option can be worth more than its European counterpart because of the possibility that it will go so deeply into the money that early exercise will be profitable.

The Effects of Cash Dividends: A Digression

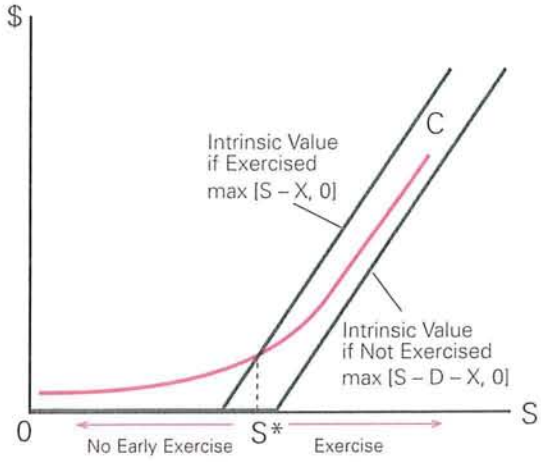
The previous sections assume that the stock underlying the equity option pays no cash dividends during the life of the option. While the exposition that follows maintains this assumption, it is clearly not universally valid. Therefore we briefly extend the option pricing model to acknowledge cash dividends.

Cash dividends do not complicate the story for a European option because it must be held to expiration, but they do require modification of the pricing of American options. Under certain circumstances it is profitable to engage in "dividend capture" strategies, which require early exercise of American options. These strategies involve buying a call option and converting it into stock in order to receive the dividend, then selling the stock. Note that since a call option is equivalent to a leveraged purchase of stocks, the same result can be obtained by borrowing money to buy the stock, then closing that position out after the ex-dividend date.

It is clear that the only incentive for early exercise for dividend capture occurs just before the stock goes

Figure 3

Early Exercise of American Call on Dividend-Paying Stock



ex-dividend. The reason is that exercise before the ex-dividend date requires payment of the strike price before the rights to the dividend are received, hence exercise before the ex-dividend date means that you pay the strike price earlier with no offsetting benefit. It would be wiser to delay the exercise until the last moment before the stock goes ex-dividend. Thus, early exercise will be prompted by dividends only *at* ex-dividend dates.

The decision at each ex-dividend date involves a simple calculation. If the option is exercised, the holder receives the subsequent stream of cash dividends but, because exercise gives only the intrinsic value of the option, the time value is lost. Hence, the present value of the dividend is received in exchange for a capital value loss. Early exercise for dividend capture is optimal only when the value of the dividends exceeds the time value of the option. The "break-even" stock price for triggering dividend capture occurs when the option is in-the-money. This is illustrated in Figure 3. The horizontal axis is the observed (*cum*-dividend) price, which includes the value of the dividend to be received. The higher kinked line shows the intrinsic value if the option is exercised early; this includes the value of the cash dividend. The lower kinked line shows the intrinsic value if the option is *not* exercised, in which case the option's value depends on the *ex*-dividend price, defined as the observed (cum-dividend) price less the value of the dividend. Thus, the vertical distance between the two intrinsic value lines is the value of the cash dividend. The red convex curve shows the call premium as a function of the *ex*-dividend price. This is the value of the call if it is not exercised.

A dividend-capture incentive to exercise the option is present when the intrinsic value upon exercise exceeds the value of the call if it is not. In that case, the dividends received exceed the value of the call sacrificed. In Figure 3 the "break-even" occurs at a stock price of S^*. Dividend capture is profitable at any stock price above S^*, and carries a net loss at any lower stock price. Note that at that break-even point the option must be in-the-money, for the observed (cum-dividend) stock price must exceed the strike price.

We see that while early exercise of an American call option is never optimal in the absence of a cash dividend, it will be optimal if the option is sufficiently in-the-money. As a result, American call options on dividend-paying stocks will have premiums above those on equivalent non-dividend stocks.

Determinants of Call Premiums

The call premium depends on several variables, and any changes in the premium must be due to changes in one or several of these variables. While calculation of the parameters describing the effect of these variables is cumbersome in a binomial model, it is very straightforward in the familiar Black-Scholes model, upon which this section is based. The comparison of the two models, and the calculation of the relevant parameters in the Black-Scholes model, is developed in Box 2.

The most prominent underlying variable is the *stock price*. As shown in Figure 1, a convex relationship exists between the call premium and the contemporaneous stock price, so changes in price affect the premium in the same direction and with increasing force. The parameters that describe this relationship are its slope, called the option's *delta*, denoted by Δ, and the rate of change in the slope, called the *gamma* and denoted by Γ.[5] Because both delta and gamma are positive, the relationship is convex. This convexity arises from the fact that at a very low stock price the option is almost certain to expire out-of-the-money, so

[5] This delta is the same as the delta in the binomial model. Both represent the change in the call premium when the stock price changes. For the binomial model, with discrete price movements, the value of delta is given by the first equation in (3).

Box 2: Relationship of the Binomial Model with the Black-Scholes Model

The binomial pricing model can be written in a way that more accurately conveys the intuition underlying it. Equation (B1.1) can be rewritten as

$$C = SN_1 - Xr^{-(T-t)}N_2 \qquad \text{(B2.1)}$$

$$N_1 = \sum_{i=0}^{T-t} q^i(1-q)^{T-t-i}[\mu^i\delta^{T-t-i}/r^{T-t}]$$

$$N_2 = \sum_{i=0}^{T-t} q^i(1-q)^{T-t-i} \quad i = max(x^*,0), \ldots, T-t$$

$$x^* = \ln(X/S\delta^{T-t})/\ln(\mu/\delta)$$

The value of N_1 is the expected present value of a dollar invested in the stock conditional on the option ending in-the money, hence the term SN_1 is the expected present value of the stock obtained if the option is exercised. The value of N_2 is the probability that the option will end in-the-money, hence the second term is the expected present value of the strike price. Therefore, the call premium is simply the expected net present value of the option, if it is worth exercising it.

Cox, Ross, and Rubinstein (1977) have shown that the binomial model converges to the more familiar Black-Scholes (1973) model when time is continuous rather than discrete, and when the probability distribution of stock prices is log-normal. The Black-Scholes pricing model for a European call option on a stock having the value for the standard deviation of its rate of price change is

$$C = SN(d_1) - Xe^{-r(T-t)}N(d_2) \qquad \text{(B2.2)}$$

$$d_1 = [ln(S/X) + (r + \tfrac{1}{2}\sigma^2)(T-t)]/\sigma\sqrt{(T-t)}$$

$$d_2 = d_1 - \sigma\sqrt{(T-t)}$$

where $N(d)$ is the probability that a standard normal random variable is less than d. The interpretation is precisely the same as that of the binomial model: The value of the call is the value of the option replicating portfolio, which consists of $N(d_1)$ shares of the stock of which the amount $Xe^{-r(T-t)}$ $N(d_2)$ is financed by borrowing at the riskless interest rate. As in the binomial case, the first term, $SN(d_1)$, is the expected present value of the stock obtained by exercising the option and $Xe^{-r(T-t)}$ $N(d_2)$ is the expected present value of the strike price paid, both expectations formed conditional on the option ending in-the-money.

The Black-Scholes model is remarkably simple, given its rather arcane foundations, and it has the advantage of being very specific about the effects of changes in parameters on call premiums. In particular, it states that the response of the call premium to changes in the parameters is

(a) Delta $(\Delta_c) = \partial C/\partial S = N(d_1)$

(b) Rho $(\rho_c) = \partial C/\partial r = (T-t)Xe^{-r(T-t)}N(d_2)$

(c) Gamma $(\Gamma_c) = \partial^2 C/\partial^2 S = N'(d_1)/[S\sigma(T-t)^{1/2}]$

(d) Vega $(\Lambda_c) = \partial C/\partial\sigma = (T-t)^{1/2}Xe^{-r(T-t)}N'(d_2)$

(e) Theta $(\theta_c) = \partial C/\partial t =$
$-Xe^{-r(T-t)}\{[\sigma/2(T-t)^{1/2}]N'(d_2) + rN(d_2)\}$

(f) Chi $(\chi_c) = \partial C/\partial X = -e^{-r(T-t)}N(d_2)$

Thus, the delta and gamma (or first and second derivatives of C with respect to S) are both positive, reflecting the convexity of the premium as seen in Figure 1. The derivative of C with respect to r (the Rho) is positive, so that a rise in r will increase the call premium. The response of C to a change in volatility is also direct, so that call premiums rise when volatility increases. The negative Theta indicates that call premiums are higher for options with longer remaining lives, and that as the life of an option shortens, its premium declines. The negative Chi says that the call premium is negatively related to the strike price.

Using put–call parity the parameters describing the put option can be expressed as follows:

$$\Delta_p = \Delta_c - 1 \qquad \Gamma_p = \Gamma_c$$

$$\rho_p = \rho_c - (T-t)Xe^{-r(T-t)} \qquad \theta_p = \theta_c - rXe^{-r(T-t)}$$

$$\Lambda_p = \Lambda_c \qquad \chi_p = \chi_c + e^{-r(T-t)}$$

any change in S has little chance of affecting the final payoff and the slope approaches zero. As the stock price falls, the option-replicating portfolio approaches a bonds-only portfolio. At a very high stock price, the option is so far in-the-money that it is unlikely to expire without value, so the slope approaches one.

As the stock price increases, the option-replicating portfolio approaches a stocks-only portfolio; that is, the option is equivalent to holding a share of common stock.

The premium on a call option will increase with the *time remaining until expiration*. The parameter describing this, the option's *theta*, measures the response of the premium to the passage of time.[6] As the time to expiration increases, the distribution of the stock price at expiration widens in response to the prospect of longer runs in "ups" and "downs." Because the payoffs of the option are truncated at zero, the investor will benefit from longer runs of "ups" but not be harmed by longer runs of "downs"; hence, a longer time to expiration will confer potential gains that offset the potential losses.

The parameter describing the effect of a change in the *riskless interest rate* is the option's *rho*. While changes in r operate through several channels (the most direct being the present value of the exercise price), the net result is a positive rho. Thus, increases in interest rates, other things equal, will raise the call premium, and the greater is the rho the stronger is the effect.

Finally, *the variability of the stock's rate of return* will also affect the value of a call option. In a binomial model, the variability can be measured by the range of stock returns, or $(\mu - \delta)$. The parameter measuring the response of the call premium to changes in variability is called the option's *vega*. Vega, also called Lambda because it is denoted by Λ, is positive because a greater range of returns improves the prospects of being in-the-money, hence raising the value of the call.

Options and Futures

We have seen that any option has a replicating portfolio of stocks and bonds, and that the option-pricing formula states the characteristics of that portfolio. We now show that futures contracts are also equivalent to a portfolio of options, stocks, and bonds. Specifically, a futures contract is equivalent to a leveraged purchase of one full share of stock combined with borrowing the present value of the futures price.

A futures contract provides a zero payoff if, at expiration, the stock price is equal to the futures price determined by the market at the time the contract is initiated. For each one-dollar deviation in stock price from the contractual futures price, a one-dollar change in payoff occurs. Thus, if the futures price at time t for delivery of a share at time T is F_t, a futures contract on common stock entered into at time t has a profit or loss of $S_T - F_t$ at expiration. Buying this futures contract and simultaneously investing the amount $r^{-(T-t)}F_t$ in bonds has a value of S_T at expiration. The value of a share of the stock owned outright at time t, S_t, will also be S_T at expiration. Because arbitrage will ensure that the amount required to buy two perfectly equivalent claims on a future share must be equal at the outset, the equilibrium requires $S_t = r^{-(T-t)}F_t$. Hence, in equilibrium

$$F_t = S_t\, r^{(T-t)} \tag{6}$$

Thus, the futures price is equal to the price of a share accumulated to the expiration date at the *riskless* rate of interest.[7] Note that the accumulation factor is the riskless rate of interest, not the expected rate of return on the stock. Note also that, just as we have seen that options are equivalent to a portfolio of stocks and bonds, futures contracts are also equivalent to a (different) portfolio of stocks and bonds. From these equivalences, others emerge. For example, a futures contract must be equivalent to a suitably constructed portfolio of options and bonds. In particular, by put–call parity, it must be equivalent to simultaneously buying a call, writing a put with the same strike price and term, and investing the difference between the strike price and the futures price in bonds.

Futures contracts are also equivalent to a portfolio of options, stocks, and bonds. Specifically, a futures contract is equivalent to a leveraged purchase of one full share of stock combined with borrowing the present value of the futures price.

[6] Theta is defined as $-(\partial C/\partial t)$, the negative of the change in the premium as the time to expiration *shortens* slightly; hence theta is positive.

[7] If the security underlying the futures contract pays a cash dividend proportional to the security price, the discount rate is the riskless rate *less* the dividend yield.

Table 4
Hedging Ratio: European Calls per Share of Stock

	Number of Ups (x)									
Day	0	1	2	3	4	5	6	7	8	9
0	−1.27									
1	−1.53	−1.16								
2	−2.09	−1.32	−1.08							
3	−3.61	−1.67	−1.17	−1.03						
4	−10.36	−2.60	−1.37	−1.07	−1.01					
5	.00	−6.49	−1.92	−1.18	−1.02	−1.00				
6	.00	.00	−4.07	−1.46	−1.05	−1.00	−1.00			
7	.00	.00	.00	−2.55	−1.16	−1.00	−1.00	−1.00		
8	.00	.00	.00	.00	−1.60	−1.00	−1.00	−1.00	−1.00	
9	.00	.00	.00	.00	.00	−1.00	−1.00	−1.00	−1.00	−1.00

Parameters: $\mu = 1.10$, $\delta = 0.9091$, $\rho = 1.02$, S = X = 50.
Note: The hedging ratio is defined as h(x, n) = −1/delta(x, n). It is the negative of the reciprocal of the change in call premium per dollar change in stock price.

II. Risk Management with Options and Futures

Just as a combination of stocks and bonds can replicate an option's price movements, options can be used to hedge against movements in stock prices. Hedges can be static or dynamic. The static hedge adopts a hedging ratio and adheres to it as the future of stock prices unfolds. Dynamic hedges adjust the hedging ratio as new information comes in. Maintaining the proper hedge requires information about the sensitivity of the option premium to changes in the stock price, that is, about the option's delta and gamma.

Dynamic Hedging

Delta hedging is a form of dynamic hedging that provides a short-term hedge against relatively small stock price movements. This hedging strategy requires computation of the *hedging ratio*, defined as minus the inverse of the delta ($h = -1/\Delta$). The hedging ratio, h, is the number of calls that must be *written* to match one long share, or the number of calls that must be purchased to match one short share. A position in calls equal to the hedging ratio will ensure that if the stock price rises (falls), the value of the call position will fall (rise) by just enough to provide a complete hedge.

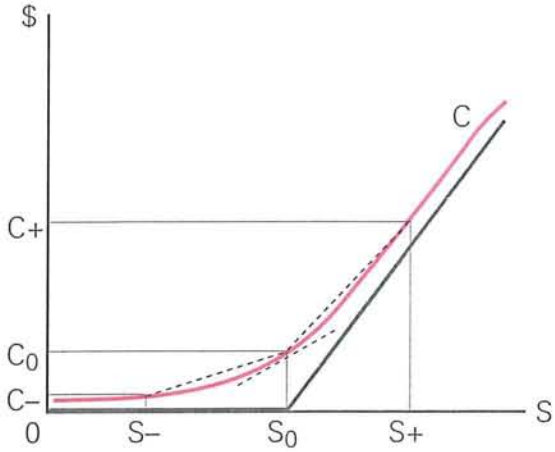
The hedging ratio implied by the hypothetical call option of Tables 1 to 3 is shown in Table 4. Each cell is the reciprocal of the associated entry in Table 2. At the outset, the investor must *write* 1.27 calls to match one long share. As time passes, the hedging ratio rises if the stock price falls, and falls if the stock price rises. As the stock price rises, the call goes deeper in-the-money and its delta increases, hence fewer calls must be written to match a dollar of stock price change.

Examination of the formula for delta reveals that it is simply the slope of the call premium–stock price relationship, as shown in Figure 1. For some derivatives, like stock index futures, the delta is constant so the gamma is zero. For these "linear" derivatives, the appropriate hedge does not change with the underlying stock price and a static hedge can be adopted. However, the convexity of the call premium–stock price relationship arising from a positive gamma means that the delta changes with stock prices. The convexity of the call premium–stock price relationship has two important implications for hedging. First, the higher the gamma, the more frequently a delta hedge must be altered as stock prices change. The reason is, of course, that the delta itself will respond more sensitively to changes in stock prices when the gamma is high. This is particularly pronounced when the option is just at-the-money, for the gamma is greatest at this point. The frequency of position adjustments can be reduced by creating both delta and gamma neutrality. Just as a position in a certain number of options can create a delta-neutral portfolio, so a portfolio that is both delta- and gamma-neutral can be constructed by using two different options on the stock.[8]

[8] Let Δ_1 and Γ_1 be the delta and gamma of one option, and Δ_2 and Γ_2 be the delta and gamma of a second option, differing in strike price or time to expiration. Let there be m and n numbers of options of each type, respectively, per share of stock held. Then the value of

Figure 4

The Role of Delta and Gamma in Dynamic Hedging



A second implication is that when gamma is high, delta hedging is more likely to fail when large stock price jumps occur. With large price changes, the optimal hedging ratio will differ from the hedging ratio designed for small changes. An example is shown in Figure 4, where the delta at stock price S_0 is shown as the dashed line tangent to the convex curve at S_0. A delta hedge can be placed by writing calls equal to the reciprocal of that delta at S_0. This will suffice to protect against small price fluctuations. Suppose, however, the stock price jumps to $S+$. The reciprocal of the slope of the line connecting C_0 and $C+$ represents the average delta, and the hedging ratio that would protect against that price jump is lower: Fewer calls need to be written to protect against large price jumps.

Suppose instead that the stock price fell to $S-$. The reciprocal of the slope of the line connecting C_0 and $C-$ will measure the correct hedging ratio. This slope is lower than the local delta at S_0, so the delta hedge will require writing fewer calls than would be necessary to protect against the fall. The value of the written calls will rise less than the decline in the stock price, leaving the delta-hedging investor exposed to stock market crashes.

the portfolio is given by $V = mC + nC + S$. A delta-neutral portfolio ($\partial V/\partial S = 0$) requires $m\Delta_1 + n\Delta_2 + 1 = 0$, while a gamma-neutral portfolio ($\partial^2 V/\partial S^2 = 0$) requires $m\Gamma_1 + n\Gamma_2 = 0$. These two equations can be solved for the values of m and n.

Thus, a hedger will be exposed to jumps in the underlying stock's price: He will write too many calls when prices jump up, and too few calls when prices jump down. There is no way out of this problem. One compromise is to use the average delta between the original position and a chosen stock price. Thus, in Figure 4 the hedging ratio for a jump to $S+$ would be the reciprocal of the line connecting C_0 and $C+$. However, this approach can increase risk exposure if the price forecast is wrong: If the price should fall rather than rise, the compromise moves the portfolio in the wrong direction.

Portfolio Insurance

Portfolio insurance is a hedging method closely related to dynamic hedging. The problem posed for portfolio insurance is to construct the equivalent of a put option on a portfolio by dynamically varying the stock-bond composition of the portfolio. Thus, a synthetic put is created by portfolio allocations that mimic the option-replicating portfolio.

Consider a financial institution with a portfolio of $100 million. Suppose that the manager wants to ensure that at the end of 10 days the portfolio is not worth less than its starting value of $100 million. A direct approach would be to buy a 10-day put on the portfolio with a strike price of $100 million. However, this might not be feasible because puts generally are not available for portfolios, because no puts are available with matching strike prices or expiration dates, or because regulation inhibits the use of derivatives. Even if an appropriate put option is available, it is more likely to be a custom-made option provided by a dealer than a standardized option traded in open markets. Hence, the put premium might be excessive, reflecting the bargaining position of the dealer.

An alternative approach is to buy "portfolio insurance" by creating a synthetic put on the portfolio. This is possible because, as we have seen, any position in calls (or puts) can be replicated using stocks and bonds. An application of portfolio insurance is demonstrated in Table 5. If the entire portfolio of $100 million is invested entirely in stocks for 10 periods, assuming the binomial stock price process with parameter values used in previous examples, the final value will range from a low of $38.554 million (if 10 "downs" occur) to a high of $259.374 million (if 10 "ups" occur). These final values are shown in the "Final Value: If 100 percent stock" row at the bottom of the table.

Suppose instead that our manager wants to estab-

Table 5
Portfolio Insurance Via Portfolio Allocation

		Number of Ups (x)										
Day		0	1	2	3	4	5	6	7	8	9	10
0	Shares (# mill)	1.573										
	stocks ($mill)	78.627										
	bonds ($mill)	25.231										
	Insured Value ($mill)	103.858										
1	Shares (# mill)	1,309.588	1,729.303									
	stocks ($mill)	59.527	95.112									
	bonds ($mill)	37.688	17.114									
	Insured Value ($mill)	97.215	112.226									
2	Shares (# mill)	957.546	1,519.449	1,854.403								
	stocks ($mill)	39.568	75.972	112.191								
	bonds ($mill)	52.989	27.949	9.888								
	Insured Value ($mill)	92.557	103.922	122.079								
3	Shares (# mill)	553.587	1,198.357	1,170.860	1,939.972							
	stocks ($mill)	20.796	54.471	94.097	129.105							
	bonds ($mill)	69.224	43.103	17.981	4.391							
	Insured Value ($mill)	90.020	97.574	112.078	133.496							
4	Shares (# mill)	193.061	768.505	1,454.602	1,863.621	1,985.487						
	stocks ($mill)	6.593	31.756	72.730	112.749	145.348						
	bonds ($mill)	82.921	61.728	31.153	9.098	1.147						
	Insured Value ($mill)	89.514	93.484	103.883	121.847	146.495						
5	Shares (# mill)	.000	308.150	1,042.934	1,700.008	1,961.156	1,999.992					
	stocks ($mill)	.000	11.576	47.406	93.500	130.515	161.050					
	bonds ($mill)	90.573	80.256	50.488	18.279	2.789	.002					
	Insured Value ($mill)	90.573	91.832	97.894	111.779	133.304	161.052					
6	Shares (# mill)	.000	.000	491.846	1,371.451	1,895.869	2,000.075	1,999.942				
	stocks ($mill)	.000	.000	20.324	68.573	114.700	146.415	177.151				
	bonds ($mill)	92.385	92.385	74.270	35.072	6.795	(.004)	.006				
	Insured Value ($mill)	92.385	92.385	94.594	103.645	121.495	146.411	177.157				
7	Shares (# mill)	.000	.000	.000	785.048	1,721.021	2,000.100	2,000.060	1,999.872			
	stocks ($mill)	.000	.000	.000	35.684	94.656	133.107	161.056	194.859			
	bonds ($mill)	94.232	94.232	94.232	62.428	16.547	(.006)	(.003)	.013			
	Insured Value ($mill)	94.232	94.232	94.232	98.112	111.203	133.101	161.053	194.872			
8	Shares (# mill)	.000	.000	.000	.000	1,253.035	2,000.000	2,000.159	2,000.001	1,999.795		
	stocks ($mill)	.000	.000	.000	.000	62.652	121.000	146.422	177.156	214.337		
	bonds ($mill)	96.117	96.117	96.117	96.117	40.278	.000	(.010)	.002	.022		
	Insured Value ($mill)	96.117	96.117	96.117	96.117	102.929	121.000	146.411	177.158	214.359		
9	Shares (# mill)	.000	.000	.000	.000	.000	2,000.000	2,000.000	2,000.254	1,999.850	1,999.762	
	stocks ($mill)	.000	.000	.000	.000	.000	110.000	133.100	161.071	194.857	235.767	
	bonds ($mill)	98.039	98.039	98.039	98.039	98.039	.000	.000	(.018)	.017	.026	
	Insured Value ($mill)	98.039	98.039	98.039	98.039	98.039	110.000	133.100	161.053	194.874	235.793	
Final	If insured ($mill)	100.000	100.000	100.000	100.000	100.000	100.000	121.000	146.410	177.160	214.360	259.370
Value:	If 100% stock ($mill)	38.554	46.651	56.447	68.301	82.645	100.000	121.000	146.410	177.156	214.359	259.374

lish a floor of $100 million at the end of 10 periods, while enjoying *all* the benefits of a 100 percent stock position if stocks increase. This means that he wants to achieve the final values shown by the "Final Value: If insured" row, computed as $max(1.10^x 0.9091^{10-x} 100, 100)$ with x "up" periods. Table 5 shows that the 100 percent stock fund will be worth at least $100 million if five or more "ups" occur in the 10 days. These "Final Value: If insured" values establish the endpoints of a dynamic option-replicating strategy. By solving backward from these final values, we can construct the dynamic path of the option-replicating portfolio of stocks and bonds that leads to these "insured" values. These computations form the cells of Table 5.

Portfolio insurance is a hedging method closely related to dynamic hedging.

Suppose that only two "ups" occur in the first nine days. Table 5 shows that the tenth day's goal is $100 million, regardless of the movement of stock prices on the tenth day. The only way this can be achieved is by putting $98.039 million into bonds, nothing into stocks, on the ninth day. Stocks are avoided because any investment in stocks will potentially break the $100 million floor. However, if six "ups" have occurred in nine days, the tenth-day goal is $146.41 million if an "up" occurs on the tenth day and $121 million if a "down" occurs. In order to achieve this, the replicating portfolio must be worth $133.1 million, all invested in stocks because there is no chance the floor will be broken.

Working all the way back we can compute the total value and composition of the portfolio required to replicate the desired final results. This leads us to an apparent paradox: In order to achieve the insurance goals, the portfolio must have a starting value of $103.86 million, an impossibility in light of the postulated $100 million starting value.

What this paradox reveals is that you cannot simultaneously enjoy the full benefits of 100 percent investment in stocks when prices increase and establish a floor that avoids price decreases. The first goal requires full investment in stocks, while the second requires some investment in bonds. The synthetic put of portfolio insurance has a hidden premium. This "insurance" premium, which should be equal to the premium that would have been paid for a standardized put option if it were available, arises from the necessary sacrifice of upside potential in order to reduce downside risk. In our example, the missing $3.86 million measures the insurance premium. Indeed, this is the premium for the synthetic put and (in the absence of transactions costs) it would also be the premium for an actual put option with a $100 million strike price.

The insurance premium in this example is 3.86 percent of the "face value" of the insurance. The institution with an initial portfolio of $100 million can do no better than establish a floor of $96.14 million, the remaining $3.86 million being the required sacrifice necessary to achieve that floor. Indeed, each number in Table 5 will be reduced by 3.86 percent to reflect the insurance premium required to keep to the desired floor. Thus, the achievable final values "if insured" will range from the floor of $96.14 million to a maximum of $249.36 million.

Portfolio Insurance with Stock Index Futures

The strategy just outlined requires frequent portfolio reallocations as stock prices change. These can entail considerable expense in terms of commissions as well as bid-asked spreads. Because transactions in stock index futures carry relatively low costs, Rubinstein (1985) suggests that an institution with a diversified portfolio might find it appropriate to use stock index futures rather than stock transactions to achieve the desired insurance.

The use of stock index futures can carry its own costs. While transactions costs are low and margin requirements are both low and often not binding,[9] there is the possibility of "basis risk" when the portfolio of stocks does not exactly match the stock index future being used. This is discussed in the next section. Even so, the net costs of the insurance can be reduced using futures.

We have seen that a futures contract should be priced so that $F = Sr^{(T-t)}$. The equivalence between futures and spot stock prices suggests a simple approach to portfolio rebalancing: Fully invest in stocks at the outset and maintain the original position in shares, using futures contracts rather than stock trades to achieve the insurance goals, and finance any profits

[9] Margin can be provided in the form of securities already held by the institution.

Table 6
Portfolio Insurance Via Stock Index Futures

		Number of Ups (x)										
Day		0	1	2	3	4	5	6	7	8	9	10
0	Insured Value ($mill)	103.858										
	Share Price ($)	50.000										
	Futures Price ($)	60.950										
	Shares (# mill)	2.077										
	Futures (# mill)	−.422										
	Shares ($ mill)	103.858										
	Bonds ($mill)	−.000										
	Actual Value ($mill)	103.858										
1	Insured Value ($mill)	97.215	112.226									
	Share Price ($)	45.455	55.000									
	Futures Price ($)	54.322	65.730									
	Shares (# mill)	2.077	2.077									
	Futures (# mill)	−.655	−.297									
	Shares ($mill)	94.417	114.244									
	Bonds ($mill)	2.798	−2.018									
	Actual Value ($mill)	97.215	112.226									
2	Insured Value ($mill)	92.557	103.922	122.079								
	Share Price ($)	41.322	50.000	60.500								
	Futures Price ($)	48.416	58.583	70.885								
	Shares (# mill)	2.077	2.077	2.077								
	Futures (# mill)	−.975	−.486	−.194								
	Shares ($mill)	85.833	103.358	125.669								
	Bonds ($mill)	6.724	.063	−3.589								
	Actual Value ($mill)	92.557	103.922	122.079								
3	Insured Value ($mill)	90.020	97.574	112.078	133.496							
	Share Price ($)	37.566	45.455	55.000	66.550							
	Futures Price ($)	43.151	52.213	63.178	76.445							
	Shares (# mill)	2.077	2.077	2.077	2.077							
	Futures (# mill)	−1.353	−.780	−.325	−.122							
	Shares ($mill)	78.030	94.417	114.244	138.236							
	Bonds ($mill)	11.990	3.157	−2.166	−4.739							
	Actual Value ($mill)	90.020	97.574	112.078	133.496							
4	Insured Value ($mill)	89.514	93.484	103.883	121.847	146.495						
	Share Price ($)	34.151	41.322	50.000	60.500	73.205						
	Futures Price ($)	38.459	46.536	56.308	68.133	82.441						
	Shares (# mill)	2.077	2.077	2.077	2.077	2.077						
	Futures (# mill)	−1.706	−1.185	−.564	−.193	−.083						
	Shares ($mill)	70.937	85.833	103.858	125.669	152.059						
	Bonds ($mill)	18.577	7.651	.025	−3.822	−5.565						
	Actual Value ($mill)	89.514	93.484	103.883	121.847	146.495						

Table 6 continued

Portfolio Insurance Via Stock Index Futures

Day		Number of Ups (x)										
		0	1	2	3	4	5	6	7	8	9	10
5	Insured Value ($mill)	90.573	91.832	97.894	111.779	133.304	161.052					
	Share Price ($)	31.046	37.566	45.455	55.000	66.550	80.526					
	Futures Price ($)	34.277	41.476	50.185	60.724	73.477	88.907					
	Shares (# mill)	2.077	2.077	2.077	2.077	2.077	2.077					
	Futures (# mill)	−1.919	−1.634	−.955	−.348	−.107	−.071					
	Shares ($mill)	64.488	78.030	94.417	114.244	138.236	167.265					
	Bonds ($mill)	26.085	13.301	3.478	−2.465	−4.932	−6.213					
	Actual Value ($mill)	90.573	91.832	97.894	111.779	133.304	161.052					
6	Insured Value ($mill)	92.385	92.385	94.594	103.645	121.495	146.411	177.157				
	Share Price ($)	28.224	34.151	41.322	50.000	60.500	73.205	88.578				
	Futures Price ($)	30.550	36.966	44.729	54.122	65.487	79.239	95.880				
	Shares (# mill)	2.077	2.077	2.077	2.077	2.077	2.077	2.077				
	Futures (# mill)	−1.957	−1.957	−1.494	−.665	−.171	−.073	−.073				
	Shares ($ mill)	58.625	70.937	85.833	103.858	125.669	152.059	183.992				
	Bonds ($mill)	33.759	21.448	8.761	−.214	−4.174	−5.648	−6.834				
	Actual Value ($mill)	92.385	92.385	94.594	103.645	121.495	146.411	177.157				
7	Insured Value ($mill)	94.232	94.232	94.232	98.112	111.203	133.101	161.053	194.872			
	Share Price ($)	25.658	31.046	37.566	45.455	55.000	66.550	80.526	97.436			
	Futures Price ($)	27.228	32.946	39.865	48.237	58.366	70.623	85.454	103.400			
	Shares (# mill)	2.077	2.077	2.077	2.077	2.077	2.077	2.077	2.077			
	Futures (# mill)	−1.997	−1.997	−1.997	−1.242	−.342	−.074	−.074	−.074			
	Shares ($mill)	53.296	64.488	78.030	94.417	114.244	138.236	167.265	202.391			
	Bonds ($mill)	40.936	29.744	16.202	3.696	−3.041	−5.135	−6.212	−7.518			
	Actual Value ($mill)	94.232	94.232	94.232	98.112	111.203	133.101	161.053	194.872			
8	Insured Value ($mill)	96.117	96.117	96.117	96.117	102.929	121.000	146.411	177.158	214.359		
	Share Price ($)	23.325	28.224	34.151	41.322	50.000	60.500	73.205	88.578	107.179		
	Futures Price ($)	24.268	29.364	35.530	42.992	52.020	62.944	76.162	92.157	111.509		
	Shares (# mill)	2.077	2.077	2.077	2.077	2.077	2.077	2.077	2.077	2.077		
	Futures (# mill)	−2.036	−2.036	−2.036	−2.036	−.808	−.076	−.076	−.076	−.076		
	Shares ($mill)	48.451	58.625	70.937	85.833	103.858	125.669	152.059	183.992	222.630		
	Bonds ($mill)	47.666	37.491	25.180	10.283	−.929	−4.669	−5.648	−6.833	−8.271		
	Actual Value ($mill)	96.117	96.117	96.117	96.117	102.929	121.000	146.411	177.158	214.359		
9	Insured Value ($mill)	98.039	98.039	98.039	98.039	98.039	110.000	133.100	161.053	194.874	235.793	
	Share Price ($)	21.205	25.658	31.046	37.566	45.455	55.000	66.550	80.526	97.436	117.897	
	Futures Price ($)	21.629	26.171	31.667	38.317	46.364	56.100	67.881	82.136	99.385	120.255	
	Shares (# mill)	2.077	2.077	2.077	2.077	2.077	2.077	2.077	2.077	2.077	2.077	
	Futures (# mill)	−2.077	−2.077	−2.077	−2.077	−1.851	−.077	−.077	−.077	−.077	−.077	
	Shares ($mill)	44.046	53.296	64.488	78.030	94.417	114.244	138.236	167.265	202.391	244.893	
	Bonds ($mill)	53.993	44.743	33.551	20.009	3.622	−4.244	−5.136	−6.212	−7.517	−9.100	
	Actual Value ($mill)	98.039	98.039	98.039	98.039	98.039	110.000	133.100	161.053	194.874	235.793	
10	Insured Value (=Actual)	98.039	98.039	98.039	98.039	98.039	100.000	121.000	146.410	177.160	214.360	259.370
	Share Price	19.277	23.325	28.224	34.151	41.322	50.000	60.500	73.205	88.578	107.179	129.687
	Futures Price	19.277	23.325	28.224	34.151	41.322	50.000	60.500	73.205	88.578	107.179	129.687

or losses on the futures contracts by lending or borrowing. Alternatively, one could fully invest in bonds at the outset and vary the positions in stocks and futures contracts to achieve the goals.

Table 6 demonstrates the first approach. An initial amount of $103.86 million is assumed so that the portfolio insurance results of Table 5 can be used as a guide. Variations in bonds and futures contracts are constructed to replicate the entire path of "insured value" outcomes in Table 5. This is done by buying futures contracts after stock price increases and selling them following a decline in prices, thereby realizing some of the futures profits. The amount of futures profits to realize is just enough so that, when invested in bonds, the value of the portfolio is equal to the insurance goal.

For example, on day zero the institution has $103.858 million to invest. It holds all of this in stocks, none in bonds, and *writes* 10-day futures contracts for 0.422 million shares. The futures price is determined as $F = 50(1.02)^{10} = \$60.95$. If the stock price goes up to $55, the value of shares on day 1 is $114.244 million and paper losses of $2.018 million occur on the futures contracts, arising from the increase in the futures price to $65.73. These futures losses are all realized and are paid for by borrowing $2.018 million. The net value of the portfolio is $112.226 million, which is the insurance goal under the plan established in Table 5. Then, in order to position itself for the second day, the institution will write 0.125 futures contracts. Thus, it will conclude the first day with a portfolio worth $112.226 million, of which $114.244 million is stock financed by borrowing $2.018 million, plus a short position of 0.125 futures.[10]

Had prices gone down on the first day, the manager would have experienced a gain of $6.628 per futures contract, for a total profit of $2.798 million. This profit would be realized and invested in bonds so that the total value of stocks and bonds would be the insurance goal of $97.215 million. The institution would then write $0.655 million of new 9-day contracts.

The portfolio insurance strategy using futures can be summarized as follows. Determine your insurance goals for each period; these are the actual portfolio values necessary to end up with the portfolio worth at least the desired floor. Then invest all your initial money in stocks and write the appropriate number of futures contracts. On each day realize enough profit or loss on your futures contract to achieve your insurance goal, investing all realized futures profits in bonds (or borrowing to finance realized futures losses). At the end of the 10-day period you will have the outcomes you selected when you undertook the portfolio insurance.

Stock Price Dynamics and Portfolio Insurance

Following the Crash of 1987, portfolio insurance became unfashionable, for two fundamental reasons. First, the dynamic interactions between portfolio insurance and stock prices suggested to some that portfolio insurance contributed to the magnitude of the Crash. This view was most forcefully presented in the Brady Commission's 1988 analysis of that event (Brady 1989). Second, institutions that had bought portfolio insurance strategies found that they were not protected from the short, but violent, free fall in stock prices.

The first criticism, that portfolio insurance magnifies stock price movements, is demonstrated in Table 5. A glance across any row (or down any column) shows that as stock prices increase (decrease), the portfolio insurer will buy more (fewer) shares. The reason for this "buy high–sell low" feature of portfolio insurance is that the need for insurance is inversely related to the stock price. An increase in stock prices reduces the likelihood that the floor will be reached, hence requiring that less money be put into riskless bonds and a correspondingly greater number of shares held. If, on the other hand, prices have been falling, the likelihood that the final result will exceed the floor is high, and the portfolio must be more heavily weighted with bonds to ensure that the floor will not be broken.

The use of futures markets as a substitute for the spot market will not eliminate the unfortunate market dynamics of portfolio insurance. While Table 6 suggests that the dynamics are absent because there are no transactions in stocks, the pressure remains but is hidden from view. The strategy underlying Table 6 has futures contracts being bought (sold) after the stock price increases (declines). As the futures price increases in response to buying pressure in that market, index arbitrageurs will find it advantageous to buy in the spot market and sell futures short, thereby transferring the pressure back to the spot market.

The second problem, failure to protect, reflected less on the merits of portfolio insurance and more on the specific circumstances of the Crash. Any insurance

[10] Of course, the institution will not buy back 0.422 futures then sell 0.125 futures. Instead, it will simply buy the difference, or 0.297 futures, leaving the remaining 0.125 short futures on the books.

program rests on assumptions about the normal range of outcomes an insurer will experience. Insurance rests on actuarial tables that work well for a pool of unrelated risks, but companies are exposed to losses from extreme outcomes, especially those that create correlated risks, like major earthquakes. Wind insurance can also fail to protect when extreme storms occur, such as Hurricane Andrew, rather than microstorms, like tornadoes.

Portfolio insurance failed during the 1987 Crash because the magnitude of the event was so extreme, because trading halts prevented execution of the insurance program, and because reported stock prices gave an incorrect signal about the state of the markets.

The Crash of '87 was an extreme outcome, and we should not be surprised that financial institutions did not find the protection they sought. The heart of portfolio insurance is the frequent adjustment of hedge positions. During a crash, a portfolio insurer would want to sell stocks and buy bonds. But, as noted in a previous article in this *Review* (Fortune 1993), this could not be done during the 1987 Crash, for a number of reasons. Trading in many stocks, as well as trading in stock index futures, was suspended for a significant portion of the time, and the backlog of unexecuted orders for those stocks that were trading was unusually long. A portfolio insurer could not maintain his program in this environment.

A problem related to this is the existence of "stale" prices in the cash market for stocks. During the Crash the *reported* levels of stock prices and stock indices exceeded the true level. This happened for two reasons, trading halts and limit orders. During a halt, the reported price is the price of the last trade, which becomes more "stale" the longer the halt. Because stock indices use these stale prices, halts give the appearance of a smaller price decline and discourage the sales that should be made to provide portfolio insurance. Probably more important were limit orders. If the specialist's book is filled with limit buy orders, a selling panic will not have its full effect on prices, because the surge of sell orders will be matched with the book, giving the appearance of a more gradual price decline. Not until the limit orders are exhausted will a free fall in prices show up.

Thus, portfolio insurance failed during the Crash because the magnitude of the event was so extreme, because trading halts prevented execution of the insurance program, and because reported stock prices gave an incorrect signal about the state of the markets.

III. Further Considerations

We have shown that "plain-vanilla" derivative instruments like equity options, index options, and index futures can be viewed as equivalent to a traditional portfolio of stocks and bonds. The analysis focused on the market risks arising from derivatives and abstracted from other types of risk. It also assumed that the statistical properties of the price of the underlying security are known. For example, the binomial pricing model assumes that prices are binomially distributed, while the Black-Scholes model assumes a log-normal distribution. In this section we address some of those loose ends.

The Limits of the Equivalence between Derivatives and Traditional Instruments

The equivalence we have demonstrated applies to a wide range of derivative products. For example, a plain-vanilla *interest rate swap* is equivalent to purchasing a fixed-rate instrument and financing it with floating-rate debt. The swap is designed so that it has a zero net present value. The swap is equivalent to buying a fixed-rate bond and financing it with a floating-rate bond. The incentive to engage in a swap is related to the comparative advantage arising from the gap between what the two parties would pay on the fixed-rate instrument and the gap on floating-rate instruments. If one company (AAA) can borrow both floating and fixed at lower rates than another (BBB), AAA has an *absolute* advantage over BBB in credit markets. Even so, AAA might have a *comparative* advantage in the fixed-rate market, while BBB has a *comparative* advantage in the floating-rate market. Hence both AAA and BBB might gain from having AAA borrow at a fixed rate and engage in a receive-fixed swap with a floating-rate borrower like BBB.

This effectively converts AAA's fixed-rate liability to a floating rate, and BBB's floating-rate liability to a fixed rate, both at more advantageous terms.[11]

To extend the metaphor, the notorious *inverse floater* is equivalent to lending at a fixed rate and borrowing a fraction or multiple of that notional value at a floating rate. A complex variation, stepped inverse bonds are equivalent to a series of forward contracts.

For some derivatives it is difficult to establish an equivalence, because no replicating portfolio, or closed form solution, can be found.

An example is the stepped inverse bond issued by the Federal National Mortgage Association (FNMA) and bought by Orange County, California, in February of 1994.[12] These bonds paid a rate of 7 percent for the first three months, then paid 10 percent minus the three-month LIBOR rate at each three-month interval until 1996 (this amounted to 5.1 percent in the first three months). In 1996 the terms changed to 11.25 percent less three-month LIBOR until maturity in 1999. This was equivalent to buying a FNMA bond due in 1996 at 10 percent and borrowing at LIBOR, for a net return of 5.1 percent in the first three months, while simultaneously engaging in a forward contract to buy a three-year 11.25 percent FNMA bond in 1996 and to sell a three-year floating rate note at three-month LIBOR.

These derivatives have a property shared by all derivatives that satisfy the equivalence property—they have replicating portfolios which allow the derivation of closed-form solutions for the derivative price. A closed-form solution means that the price can be expressed as a function of relevant variables and parameters. For example, the Black-Scholes model states that a European call option is equivalent to a fractional share of the underlying security plus borrowing a fractional share of the exercise price; this is a closed-form solution.

However, for some derivatives it is difficult to establish an equivalence, because no replicating portfolio, or closed-form solution, can be found. This can occur for several reasons. One reason might be that the standard assumptions about the probability distribution of final payoffs do not apply. Typically, this means that there is no precise replicating portfolio and valuation must be done by numerical methods. An example is a *lookback option*, for which the exercise price depends upon the history of the underlying security's price. In one form of lookback option, the Asian-style option, the strike price is the (arithmetic) average price of the underlying security over the life of the option. In this case the distribution of payoffs is not normal even though the returns on the underlying security might be normally distributed.[13] Closed-form solutions based on the normal distribution will not apply, and numerical simulation methods are the only alternative approach to valuation. Hence, we cannot treat an Asian option as equivalent to a portfolio of stocks and riskless bonds. We can only recognize the fuzzy correspondence between the lookback option and a portfolio.

Basis Risk

Another reason for absence of a replicating portfolio is transactions costs. These can make it expensive to engage in the portfolio-option arbitrage which allows a closed-form solution. Often this gives rise to *basis risk*. The problem of basis risk arises when an option or futures contract based on an index of securities is used to hedge a portfolio that is not precisely mimicked by the derivative. For example, our financial institution's portfolio insurance scheme employed stock index futures to adjust the effective portion of the portfolio devoted to stocks. If the institution was an index fund holding, say, the S&P 500 portfolio of

[11] For example, if AAA can borrow at 10 percent fixed and LIBOR + 1 percent floating, while BBB can borrow at 12 percent fixed and LIBOR + 2 percent floating, then AAA has a comparative advantage as a fixed-rate borrower. Suppose AAA borrows fixed at 10 percent and engages in a fixed-rate swap in which it pays LIBOR + 1.5 percent floating in exchange for 11 percent fixed. BBB, of course, borrows floating at LIBOR + 2 percent and takes the other side of the swap, paying 11 percent fixed and receiving LIBOR + 1.5 percent floating. The net effect is that AAA has a floating-rate liability at LIBOR + 0.5 percent and BBB has a fixed-rate liability at 11.5 percent. Both parties are better off.

[12] In a structured note, a government agency issues a bond whose rate of return is set according to a specified relationship with a short-term interest rate, with the payments reset at the interval of the short rate. The relationship between the rate paid and the short rate can be direct or inverse, fractional or multiple. By selling offsetting structured notes, the agency's obligation can be equivalent to a fixed-rate bond.

[13] An Asian call option based on the average price has the payoff $max(S - S_{avg}, 0)$. Even though S might be log-normally distributed, S_{avg} is not. Indeed, $S - S_{avg}$ has no known distribution.

stocks, the use of an S&P 500 futures contract involves no basis risk. But in most cases the stock portfolio will differ in composition from the portfolio upon which the futures contract is based. As a result, the correlation between the derivative price and the value of the stocks in the portfolio is imperfect.

Basis risk clearly means that hedges of the type outlined above are imperfect. However, stocks have been shown to have a significant common factor (called the market factor) in their price movements. Because idiosyncratic risks attached to specific stocks can largely be diversified away, the major source of price variation is the market factor. In short, funds do not have to be index funds to be highly correlated with stock market movements, and the higher this correlation the smaller is the basis risk.

Counterparty Risk

Counterparty risk is the prospect that the counterparty to a derivative transaction might default. For example, the writer of a call option is obligated to deliver shares in the event that the call is in-the-money and the holder exercises it. If the writer reneges, and if no insurance pool or other means is available to enforce the contract, the value of the holder's call is reduced or disappears.

Counterparty risk is a relatively small problem in the exchange-traded options and futures we have focused on here. The reason is that the transaction is not with a specific counterparty, but with a clearing house. The functions of the clearing house are to determine to which writer an exercised contract is assigned, to receive payments for and make transmittal of underlying securities, and to ensure that all contracts are honored. Any default is with the clearing house, which has adequate resources to meet the obligations of the contract. The clearing house provides an extremely important function in the formation of actively traded markets for standardized contracts, for it removes the specific names on the contract from each party's consideration. In the absence of the clearing house, any buyer of an option contract, or participant in a futures contract, would require detailed information on the financial position of the counterparty, a requirement that would inhibit use of the instruments.

Counterparty risk is considerably more important in the over-the-counter (OTC) markets for derivatives. These dealer markets create custom-made contracts between parties, which have less liquidity (that is, they are more difficult to reverse) and for which no guaranteeing agency is present to ensure payments. Thus, the pricing of OTC products ranging from plain-vanilla interest rate swaps to exotica like diff swaps[14] involves counterparty risk, which affects the value, or terms, of the derivative. Counterparty risk can be reduced in a variety of ways, from compensatory interest rates (in the form of a premium paid over LIBOR or over Treasury bonds) through credit enhancements (for example, collateral) to netting agreements between the parties. Recently, efforts have been made to organize a clearing-house arrangement for OTC derivatives, although none has been established as of this writing.

IV. Summary and Conclusions

This article demonstrates that such simple derivatives as exchange-traded options can be easily understood if one is just willing to spend some time at it. Many, but not all, of the new derivatives are, in fact, old instruments in new clothing. As such, they may represent more complicated ways to speculate or to hedge, but in most cases they can be understood as equivalent to traditional instruments. Indeed, one approach to evaluating the risk exposure of financial institutions holding derivatives is to convert them to their equivalence in more traditional financial instruments.

In order to understand the equivalence between many derivatives and traditional portfolios of stocks and bonds, we first explain the technology of derivatives. This is done for the simplest forms of derivatives—plain-vanilla equity options, stock index options, and stock index futures. We show that these derivatives are equivalent to an underlying portfolio of stocks and bonds. Hence our subtitle "A Rose by Any Other Name . . . ." This equivalence is illustrated in several ways. We first demonstrate the equivalence using a simple formal model of option price determination, the *binomial option pricing model*, in which options premiums are directly derived from the prices of stocks and bonds. Because the binomial model serves as a first approximation to option premiums described by more sophisticated theory (for example, the Black-Scholes model), it is a particularly fruitful starting point.

[14] A diff swap involves payment of a foreign interest rate by one party in exchange for a U.S. interest rate by the other, all payments in U.S. dollars. Hence, it is equivalent to borrowing at the U.S. rate to buy foreign securities, combined with an exchange rate guarantee.

After discussing the pricing of stock options and stock futures, and their equivalence to stocks and bonds, we then turn the problem on its head and show how stocks, bonds and futures can be used to mimic options; that is, traditional stock-bond portfolios can be used to construct "synthetic" options. Thus, in the second section we focus on aspects of risk management using synthetic options constructed by taking positions in traditional instruments. Our particular interest is in dynamic hedging, and in one of its more interesting manifestations, portfolio insurance. We point out why portfolio insurance seemed such a promising strategy, and what led to its decline after the Crash of 1987.

No metaphor precisely fits all circumstances, and our analogy between traditional instruments and exchange-traded options does not apply to all derivative securities. We discuss several circumstances in which there is no replicating portfolio and, therefore, the equivalence between a derivative and traditional instruments breaks down. Among these are basis risk, counterparty risk, and discontinuities in the price of the underlying security. In these cases, derivative securities do introduce something new because they do not precisely replicate the movements in prices of traditional instruments. These situations are most often found in over-the-counter derivatives, though they can also apply to exchange-traded derivatives.

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